Source Current Polarization Impact on the Cross-Polarization Definition of Practical Antenna Elements: Theory and Applications

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***Abstract*—With the growing interest in polarization diversity in communications and radar systems, the use of Ludwig's second and third definitions have become controversial among scientists and antenna engineers. Therefore, this manuscript is an attempt to clarify some of the ambiguity and confusion caused by these definitions. A detailed comparison of Ludwig's 2nd and 3rd definitions of cross-polarization, as applied to linearly polarized antennas, was performed. The results show that, in the diagonal plane, Ludwig's 2nd definition leads to a lower cross-polarization level than the 3rd definition for x- or y-polarized current sources. For a Huygens source, by Ludwig's 3rd definition, the radiation pattern has a lower cross-polarization level than that obtained by Ludwig's 2nd definition. For some applications, the antenna is usually placed in the y-z plane. Therefore, new polarization bases are proposed according to which source is used as a reference, and also on how this source is oriented in the y-z plane.**

To complement the theoretical framework demonstrated in this contribution, and to provide readers with a better and simpler understanding of the cross-polarization definition, the analysis of several practical antennas for diverse applications was presented. Numerical and measured radiation patterns of wire and printed dipoles, rectangular patch, pyramidal horn, and open-ended rectangular waveguide antennas were investigated according to the polarization formulations presented in this paper. In addition, a dual-polarized element and dual-polarized active phased array at broadside were utilized to generalize the application.

***Index Terms*—Radar systems, polarization diversity, cross-polarization, Ludwig's definitions, source current polarization, far-field polarization, phased array.**

I. Introduction

In applications such as satellite communications, radar systems, and remote sensing, it is very important to make more efficient use of available bandwidth to effectively increase channel capacity. Instead of a spatial diversity approach, the use of a polarization diversity provides two communication channels for each frequency band. For this reason, interest has been increased in the polarization purity of antenna patterns and cross-polarization reduction [1]. In the polarization diversity approach, two independent signals using the same frequency band can be transmitted over a single link. In such systems, isolation between channels depends on suppression of the cross-polarization. High cross-polarization levels will degrade the quality of the orthogonal signals by mutual interference [1-4]. Achieving pure polarization with the lowest possible levels of cross-polarization is very important for these applications. However, designing a system with extremely low cross-polarization levels in the coverage region is not easy, although the orthogonally-polarized channels are theoretically assumed to be completely isolated.

For polarimetric phased array weather radar, steering the co-polar beam away from broadside direction or the principal planes will dramatically change the cross polarization level along the boresight direction, with the highest value in the diagonal plane. For this reason, obtaining very low cross polarization and high port isolation between the orthogonal antenna ports (H- and V-polarizations) over the whole scanning range is the major challenge for any polarimetric weather radars [5].

In 1973, the major paper concerning definitions of co- and cross-polarization was published by Ludwig [6]. He discussed and presented the definitions of co- and cross-polarization as applied to linearly polarized antennas. All Ludwig's definitions are essentially the same in the principal planes, but they seriously disagree in off-broadside directions along non-principal planes. In Knittel's commentary on Ludwig's paper [7], the author mentioned that the Ludwig 3 definition cannot be the standard definition of the cross-polarization, and it is not optimal for electric and magnetic dipoles. According to the Ludwig 3 definition, both dipoles would have significant levels of cross polarization out of the principal planes with highest value in the diagonal plane ($\phi = 45°$, $\theta = 45°$). Also, a Huygens source would have no cross-polarization under the Ludwig 3 definition. In [8], a θ-dependence, not involved in the original Ludwig 3 definition, was introduced to generalize the Ludwig's third definition. Ambiguity and confusion regarding the use of the most meaningful description of cross polarization have been caused due to the controversy surrounding these definitions, and not much work has been done to identify and clarify them. In this paper, the controversy is addressed by attempting to clarify Ludwig's definitions using source current polarization and its relation to the far field polarization of different linearly-polarized reference sources positioned in different orientations. In addition, as an extension of Ludwig's definitions, co- and cross-polarization definitions are provided for reference sources positioned in the spherical coordinate system with different configurations.

The coordinate system of an anechoic chamber has a different configuration than the standard coordinate system used in theory and in weather radar applications. In these configurations, an antenna is lying in the y-z or x-z plane. Therefore, well-known definitions of co- and cross-polarization, which were derived assuming the reference source lying in the x-y

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plane, need to be properly extended to provide much more accurate analytical expressions to characterize co- and cross-polarization unit vectors.

The newly developed definitions, as well as Ludwig's definitions, are applied to linearly polarized different practical antenna elements to clarify the cross-polarization definition, and to discuss the proper definition for different applications. The cross-polarization levels of a wire and printed dipole, rectangular microstrip patch, pyramidal horn, and open-ended rectangular wave guide have been used to illustrate Ludwig's definitions. In addition, dual-polarized 4x4 antenna array has been used for the same purpose.

Although cross-polarization definitions are provided by Ludwig, there are no detailed mathematical derivations. Therefore, the antenna community is still confused when following the formulas presented by Ludwig. Primarily for educational purposes, another objective of this paper is to provide a detailed formulation of Ludwig's definitions of the co- and cross-polarization using critical notes found in his paper [7].

The rest of this paper is organized as follows: Ludwig's definitions of co- and cross-polarization are reviewed in more detail in Section II. Next, Section III presents a detailed description of the relationship between the source current polarization and co- and cross-polarization components of the radiation pattern in the far field region. In Section IV, the extended cross-polarization definitions of an antenna lying in the y-z plane are presented taking into account the effect of mechanical elevation tilt. In Section V, the HFSS simulated and measured results of co- and cross-polarization components of different antennas are conducted for the purpose of verifying the definitions presented in the paper. Finally, Section VI summarizes all derived work and concludes the paper.

II. CROSS-POLARIZATION DEFINITION

Polarization characteristics of the electromagnetic fields radiated by an antenna are one of the main factors that must be considered in the antenna design. In general, depending on the type of application, the antenna is designed to operate in a certain mode of polarization that typically varies from linear to circular. However, purity of the desirable polarization within the co-polar beam is required. This condition is normally satisfied for antennas with a very high directivity in which the cross-polarization level is sufficiently low within a narrow angular sector around the broadside direction. However, for non-directive radiating elements like those used in array antennas, the cross-polarization level is significantly high over a wide angular sector. The presence of cross-polarization in this angular sector of the radiation pattern undesirably impacts the antenna performance, because cross-polarized components are radiated at the expense of desirable co-polarized components. The energy trapped in the cross-polarization component is considered a loss in the total input energy, which affects antenna efficiency. Hence, efficient antennas are designed to minimize cross-polarization levels.

For any practical antenna, discrepancies among the cross-polarization levels using Ludwig's definitions can be observed in the region away from the broadside direction especially in the non-principal planes. These discrepancies are serious in the angular region about 5° to 80° from the antenna broadside with the maximum value at approximately $\phi = 45^\circ$ [9].

In the spherical coordinate system, the unit vectors $\hat{\theta}$ and $\hat{\phi}$ are commonly used to represent the theoretical and measured fields radiated by an antenna. The electric field components at any observation point in the far-field region of the antenna are specified by the angles θ and ϕ. In the principal planes and at broadside, both of these spherical unit vectors are aligned to cartesian unit vectors. However, in off-broadside directions along non-principal planes, the co- and cross-polarization vectors depend on how the polarization basis is defined. Both components will be coupled with each other when scanning off-broadside and off-principle planes. The angular spatial relationship between the field components in the off-broadside angle and along non-principal planes is a matter of geometric projection of the electric field components [5].

In electromagnetics and antenna theory, different coordinate systems are used to describe the radiating sources and their radiated waves. The radiating sources are usually described in terms of a cartesian coordinate system. On the other hand, a spherical coordinate system, with the same cartesian origin, is used to describe the far-field waves radiated by these sources. However, some ambiguities and confusion in the interpretations and applications of appropriate co- and cross-polarization definitions are created because both coordinate systems uses the same origin. Consequently, one definition of cross polarization, universally accepted, does not exist [5], [6].

Most antennas are typically designed to work in a certain polarization mode. However in reality, these antennas, in addition to the designed polarization mode, have radiation leakage in the perpendicular polarization direction. Hence, the antenna simultaneously has two radiation patterns, co- and cross-polarization. Therefore, the term *cross-polarization* arises because there is no antenna perfectly polarized in a single mode. The IEEE standard definition of cross-polarization is "the polarization orthogonal to a specified reference polarization" [10]. Unfortunately, this definition does not define the direction of the reference polarization, and then leads to ambiguity and confusion in the use of the appropriate definition of cross-polarization. For example, the right hand circular polarization is the cross-polarization for the left hand circular polarization, and the vertical polarization is the cross-polarization of an antenna horizontally polarized, and vice-versa. For circular polarization, the standard definition is sufficient and adequate, but for linear or elliptical polarization, the direction of the reference (co) polarization still needs to be defined. The cross-polarization level of an antenna is defined as the peak value of the cross-polarization radiation pattern relative to the peak value of the co-polarization radiation pattern. The cross-polarization level is usually calculated in the *E*-, *H*-, and *D*-planes with the highest value in the *D*-plane.

Co- and cross-polarization components of the radiating source under consideration are usually defined by comparing them to a reference source [10]. The co-polarization component of a given antenna is defined to be the field component that is parallel to the reference source field, and the cross-polarization component is the orthogonal component. These

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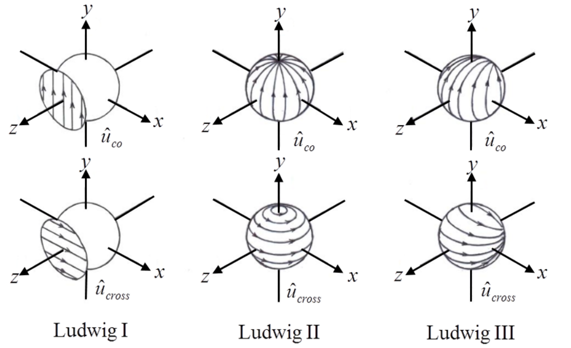

Fig. 1: Definitions of co-polarization and cross-polarization for the three definitions of Ludwig [6].

components can be expressed by deriving unit vectors $\hat{u}_{co}$ and $\hat{u}_{cross}$ of a reference source such that the dot product of these unit vectors with the electric field components of a given antenna in the far field defines the co- and cross-polarization components, respectively. These components, at a given observation angle specified by the spherical coordinate angles (θ, ϕ), are given by

$$E_{co} = \bar{\mathrm{E}} \cdot \hat{u}_{co} \tag{1}$$

$$E_{cross} = \bar{\mathrm{E}} \cdot \hat{u}_{cross} \tag{2}$$

where $\bar{\mathrm{E}}$ is the electric field vector of the given antenna, $\hat{u}_{co}$ and $\hat{u}_{cross}$ are unit vectors defined not only based on which source has been chosen as the reference, but also on how this reference is oriented.

Ludwig, in [6], discusses and presents three alternative definitions of the co- and cross-polarization. These definitions, named the 1st, 2nd, and 3rd Ludwig's definitions, are used either implicitly or explicitly in the literature. The first definition is defined according to the reference field considered as a plane wave. The second is defined by the radiated E-field from an electric dipole. Ludwig's third definition is defined by the E-field radiated by a y-polarized Huygens source. According to [6], the co- and cross-polarization of an antenna, linearly polarized, can be defined in three ways:

Ludwig 1. Unit vectors of a rectangular coordinate system coincide with co- and cross-polarization unit vector directions [5], [6]. As shown in Fig.1, the electric field vector is projected onto the $\hat{x}$ and $\hat{y}$ vectors lying in the aperture plane.

$$\hat{u}_{co} = \hat{y} = \sin\theta \sin\phi \ \hat{r} + \cos\theta \sin\phi \ \hat{\theta} + \cos\phi \ \hat{\phi} \tag{3}$$

$$\hat{u}_{cross} = \hat{x} = \sin\theta \cos\phi \ \hat{r} + \cos\theta \cos\phi \ \hat{\theta} - \sin\phi \ \hat{\phi} \tag{4}$$

In most of antenna applications, using this definition leads to inaccuracies because the fields radiated by any antenna in the far region are tangent to the surface of a sphere centralized at the field source. The polarization of the radiated fields varies as the observation angle moves away from broadside. Therefore, Ludwig 1 is fundamentally not the appropriate definition for these applications. However, the first definition is the proper choice to describe source current polarizations [6].

Ludwig 2. Spherical unit vectors, tangential to a spherical surface, are used to represent the unit vector directions of co and cross polarizations. Co- and cross-polar field vectors, corresponding to the θ and ϕ directions of a perfectly linearly polarized antenna are shown in Fig. 1. For a y-polarized infinitesimal dipole, the Ludwig 2-I definition is presented by

$$\hat{u}_{co} = \frac{\sin\phi \cos\theta \ \hat{\theta} + \cos\phi \ \hat{\phi}}{\sqrt{1 - \sin^2\phi \ \sin^2\theta}} \tag{5}$$

$$\hat{u}_{cross} = \frac{\cos\phi \ \hat{\theta} - \sin\phi \cos\theta \ \hat{\phi}}{\sqrt{1 - \sin^2\phi \ \sin^2\theta}} \tag{6}$$

If the same dipole polarized in the x-direction, the Ludwig 2-II definition is presented by

$$\hat{u}_{co} = \frac{\cos\phi \cos\theta \ \hat{\theta} - \sin\phi \ \hat{\phi}}{\sqrt{1 - \cos^2\phi \ \sin^2\theta}} \tag{7}$$

$$\hat{u}_{cross} = \frac{\sin\phi \ \hat{\theta} + \cos\phi \cos\theta \ \hat{\phi}}{\sqrt{1 - \cos^2\phi \ \sin^2\theta}} \tag{8}$$

Eqs. (5-8) show that the co- and cross-polarization components of a perfect current source are not orthogonal to others of the same source rotated 90°, except in the principal planes and at the broadside direction [6]. This is because the coordinate system that defines the co- and cross-polarization components of the radiated field cannot be rotated. The dot product of the corresponding co- and cross-unit vectors in these equations (neglecting unimportant sign changes) is not zero in all directions, and it is as follows

$$\hat{u}^{L2-I} \cdot \hat{u}^{L2-II} = \frac{\cos\phi \sin\phi \sin^2\theta}{\sqrt{\cos^2\theta + 0.25 \sin^4\theta \sin^2 2\phi}} \tag{9}$$

Eq. 9 shows the nonorthogonality between two perfect patterns rotated 90° with respect to each other. Because of this property, there are two cases of Ludwig 2, named as Ludwig 2-I and Ludwig 2-II definitions, based on the polarization direction of the antenna. For dual-polarized antennas, this definition will result co- and cross-polarization radiation patterns that are not simple versions of the one another simply rotated 90°. However, for Ludwig first and third definitions, interchanging the co- and cross-polarization field components corresponds to rotating the reference source 90° about the z-axis (neglecting sign changes).

Ludwig 3. The co- and cross-polarization definition of Ludwig 3 corresponds to "what one measures when antenna patterns are taken in the usual manner". It is not easy to formulate this definition in terms of simple coordinate system unit vectors as explained in [6]. It is normally used with feed systems and reflector antennas and is widely used in anechoic chamber measurements. As shown in Fig. 1, the Ludwig 3 unit vectors can be obtained by rotating the θ and ϕ unit vectors about the radial direction by the angle ϕ as follows

$$\hat{u}_{co} = \sin\phi \ \hat{\theta} + \cos\phi \ \hat{\phi} \tag{10}$$

$$\hat{u}_{cross} = \cos\phi \ \hat{\theta} - \sin\phi \ \hat{\phi} \tag{11}$$

An ideal Huygens source, composed of orthogonal electric and magnetic currents placed along the y- and x-axis, respectively, is used as a reference to derive the Ludwig 3 equations. Therefore, the Huygens source is considered an

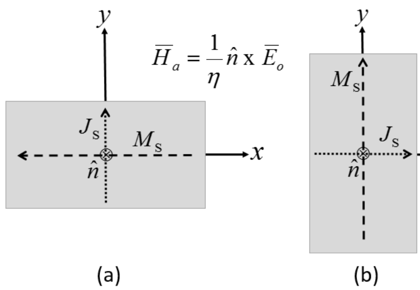

Fig. 2: Aperture antenna with a Huygens source polarized in a) y-direction b) x-direction.

ideal electromagnetic source that generates a radiation pattern with orthogonal electric fields in any beam direction and zero cross-polarization everywhere. The co- and cross-polarization components of the radiation pattern, using this definition, can be transformed into each other by a global rotation. Interchanging these equations in any direction in the far field space corresponds to a 90° rotation of the Huygens source.

For a Huygens source, the aperture tangential electric and magnetic fields are related by the uniform plane wave relationship at all points over the aperture as illustrated in Fig. 2. This condition is not easily satisfied for practical aperture antennas. Therefore, the Huygens source should be considered an approximation. However, it is approximately valid for corrugated and dual-mode horn antennas with a large aperture, but only over a small bandwidth. The high impedance side walls, implemented with corrugations, reduce the cross-polarization level because of the highly symmetric field distribution over the horn aperture. Such a source is widely used as a feed of the parabolic reflector, which theoretically has a radiation pattern with no cross-polarization. On the other hand, antenna with very small aperture compared to the wavelength like slot antenna can be considered as a magnetic dipole.

Ideally, a symmetric field distribution over the antenna aperture, with respect to the principal planes, contributes to zero cross-polarization in the symmetry planes and the boresight direction. However, the cross-polarization level dramatically increases if moved out from the symmetry planes or away from the boresight direction. For any linearly polarized antenna, the cross-polarization pattern takes the form of four lobes, with peaks located in the diagonal plane and extremely low values along the principal planes, because of the phase inversions between any two adjacent quadrants.

III. Source current polarization

Unfortunately, Ludwig's definitions of the co- and cross-polarization of an antenna, despite their popularity and wide range of applications, have not received much attention. Ludwig developed a set of classic equations that were included in [6]. While these equations have been used as a reference in many books and papers, it has not been clearly documented how they were obtained, which references were used, and in which orientations those references were positioned. Therefore, in the antenna and radar communities there still doubt when following the derivation in Ludwig's paper. This section discusses how to obtain and understand the co- and cross-polarization vectors of the far field, and their relation to the reference radiating source polarization (type and orientation).

This relationship can be simply obtained by using the *current distribution* method for either wire or aperture antenna. This method with help of the *field equivalence principle* is one of the common techniques used to calculate co- and cross-polarization performance of an antenna [11-12], in which the aperture fields become the sources of the radiated fields at far observation points. In this paper, the current distribution method was used for calculating the θ and ϕ components of the radiated field. The expressions of these components derived in the Appendix are expressed in terms of the source current polarization of an antenna.

As shown in Fig. 2 in [6], a given antenna is placed in the x-y plane with the z-axis normal to the antenna. The polar angle θ is measured from a fixed zenith direction (z-axis) and azimuth angle ϕ is measured from the x-axis to the orthogonal projection of the radial distance r on the x-y plane.

For simplicity, an infinitesimal electric or magnetic dipole oriented along the x- or y-axis are considered

$$\mathrm{J}_s = \hat{x}J_0\delta(x') \qquad \text{or} \qquad \mathrm{J}_s = \hat{y}J_0\delta(y') \tag{12}$$

$$\mathrm{M}_s = \hat{x}M_0\delta(x') \qquad \text{or} \qquad \mathrm{M}_s = \hat{y}M_0\delta(y') \tag{13}$$

The total electric field of an electric dipole polarized in the x- or y-axis (12), with (37-40) in the Appendix, respectively, is given by

$$E_t \simeq k_e(\cos\theta\cos\phi\ \hat{\theta} - \sin\phi\ \hat{\phi}) \tag{14}$$

$$E_t \simeq k_e(\cos\theta\sin\phi\ \hat{\theta} + \cos\phi\ \hat{\phi}) \tag{15}$$

where $k_e = -\frac{j\eta\beta J_o\ e^{-j\beta r}}{4\pi r}$

Using (13) and (41-44) in the Appendix, for a magnetic dipole polarized in x- or y-direction, the total electric field, respectively, is presented by

$$E_t \simeq -k_m(\sin\phi\ \hat{\theta} + \cos\theta\cos\phi\ \hat{\phi}) \tag{16}$$

$$E_t \simeq k_m(\cos\phi\ \hat{\theta} - \cos\theta\sin\phi\ \hat{\phi}) \tag{17}$$

where $k_m = -\frac{j\beta M_o\ e^{-j\beta r}}{4\pi r}$

On other hand, a Huygens source polarized in the x- or y-axis is given by the sum of two orthogonal sources (one electric infinitesimal source J_0, and one magnetic infinitesimal source M_0, where $M_0 = \eta J_0$). This source, as shown in Fig. 2, polarized in the x- and y-direction, respectively, can be represented as follows

$$\mathrm{J}M_{\text{Huygens}} = \hat{x}J_0\delta(x') + \hat{y}M_0\delta(y') \tag{18}$$

$$\mathrm{J}M_{\text{Huygens}} = \hat{y}J_0\delta(y') - \hat{x}M_0\delta(x') \tag{19}$$

Using a x- or y-polarized Huygens source given respectively in (18) and (19), and (30-35) in the Appendix, the total electric field can be given by

$$E_t \simeq k_e(1+\cos\theta)(\cos\phi\ \hat{\theta} - \sin\phi\ \hat{\phi}) \tag{20}$$

$$E_t \simeq k_e(1+\cos\theta)(\sin\phi\ \hat{\theta} + \cos\phi\ \hat{\phi}) \tag{21}$$

Now, Eqs. (1) and (2) are used such that the dot product of the unit vectors $\hat{u}_{co}$ and $\hat{u}_{cross}$ of Ludwig's definitions

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(5-8) and (10-11), and the unit vector of the total electric field that is radiated by an infinitesimal electric dipole (14-15) and an infinitesimal magnetic dipole (16-17) polarized in the x- or y-axis, will define the contributions of these currents to the cross-polarization in the far field patterns. These contributions to the cross polarization are summarized in Table I (ignoring unimportant sign changes). It can be seen that the y-polarized electric and magnetic current patterns contain no cross-polarization according to the Ludwig 2-I definition. However, by using the Ludwig 2-II definition, the pattern of the x-polarized electric and magnetic currents has zero cross-polarization. It is apparent that the dominant cause of cross-polarization is the x-polarized source current according to Ludwig 2-I, and the y-polarized source current according to Ludwig 2-II. The y-polarized source current by Ludwig 2-I and x-polarized source current by Ludwig 2-II are the co-polarization currents. If the third definition is used, the radiation patterns of the electric and magnetic currents, oriented along the x- and y-axis, would have no cross-polarization in the principal planes, and significant cross-polarization in nonprincipal planes and far away from broadside.

TABLE I: Electric and magnetic source current contributions of an infinitesimal dipole to far-field patterns cross-polarization

Definition	Direction	I^e	I^m
Ludwig 2-I	i_x	$\frac{\sin\phi\cos\phi\sin^2\theta}{\sqrt{F}}$	$\frac{\sin\phi\cos\phi\sin^2\theta}{\sqrt{F}}$
	i_y	0	0
Ludwig 2-II	i_x	0	0
	i_y	$\frac{\sin\phi\cos\phi\sin^2\theta}{\sqrt{F}}$	$\frac{\sin\phi\cos\phi\sin^2\theta}{\sqrt{F}}$
Ludwig 3	i_x	$\frac{\cos\phi\sin\phi(1-\cos\theta)}{\sqrt{1-\cos^2\phi\sin^2\theta}}$	$\frac{\cos\phi\sin\phi(1-\cos\theta)}{\sqrt{1-\cos^2\phi\sin^2\theta}}$
	i_y	$\frac{\cos\phi\sin\phi(1-\cos\theta)}{\sqrt{1-\sin^2\phi\sin^2\theta}}$	$\frac{\cos\phi\sin\phi(1-\cos\theta)}{\sqrt{1-\sin^2\phi\sin^2\theta}}$

$$F = (1-\sin^2\phi\sin^2\theta)(1-\cos^2\phi\sin^2\theta)$$

Similar procedure is followed if the Huygens source (20-21) is used; the only difference is that a combination of two currents J_x and M_y or J_y and M_x given by (18) and (19), respectively, is used to calculate the contributions of the Huygens sources to the cross-polarization in the far field patterns as shown in Table II (ignoring unimportant sign changes). The Ludwig 3 definition with a Huygens source has a radiation pattern with no cross-polarization over all the space as show in Table II. However, the Ludwig 2-I and 2-II definitions, as applied to a Huygens source, produce radiation patterns with significant levels of the cross-polarization in off-broadside directions along nonprincipal planes. The antenna orientation causes the exchange between co- and cross-polarization equations derived above. This exchange was taken into consideration in the formulas summarized in the tables.

IV. Extended cross-polarization Definition

In polarimetric weather radar applications, a planar phased array antenna is usually located in the y-z plane in the

TABLE II: Huygens source current contributions to far-field radiation patterns cross-polarization

Definition	J_y & M_x	J_x & M_y
Ludwig 2-I	$\frac{\sin\phi\cos\phi(1-\cos\theta)}{\sqrt{1-\sin^2\phi\sin^2\theta}}$	$\frac{\sin\phi\cos\phi(1-\cos\theta)}{\sqrt{1-\sin^2\phi\sin^2\theta}}$
Ludwig 2-II	$\frac{\sin\phi\cos\phi(1-\cos\theta)}{\sqrt{1-\cos^2\phi\sin^2\theta}}$	$\frac{\sin\phi\cos\phi(1-\cos\theta)}{\sqrt{1-\cos^2\phi\sin^2\theta}}$
Ludwig 3	0	0

coordinate system due to radar always assume precipitations in the x-z or y-z plane. This is different from the assumption used in Ludwig's definitions, in which the antenna is located in the x-y plane. Consequently, a new polarization basis needs to be defined following the same procedure used in Ludwig's definitions. In radar applications, vertical (V) and horizontal (H) polarization bases are the most commonly used. The horizontal axis (y-axis) is parallel to the ground and the vertical (z-axis) is parallel to gravity. For an infinitesimal electric dipole oriented vertically in the z-axis, the electric field will be directed in the θ direction with no cross-polarization as shown in Table III. If, on the other hand, an infinitesimal magnetic dipole is oriented vertically in the z-axis, the electric field will be directed in the ϕ direction with no cross-polarization. However, if the same infinitesimal dipoles (electric/magnetic) are horizontally oriented in the y-axis, the electric fields will have both θ and ϕ components. Therefore, the co- and cross-polarization components can be calculated using the θ and ϕ components as shown in Table III.

TABLE III: Co- and cross-polarization unit vectors of far-field radiation patterns

		$\hat{u}_{co}$	$\hat{u}_{cross}$
i_z	I^e	$\hat{\theta}$	$\hat{\phi}$
	I^m	$\hat{\phi}$	$\hat{\theta}$
i_y	I^e	$\frac{\cos\theta\sin\phi\hat{\theta}+\cos\phi\hat{\phi}}{\sqrt{1-\sin^2\phi\sin^2\theta}}$	$\frac{\cos\phi\hat{\theta}-\cos\theta\sin\phi\hat{\phi}}{\sqrt{1-\sin^2\phi\sin^2\theta}}$
	I^m	$\frac{\cos\phi\hat{\theta}-\cos\theta\sin\phi\hat{\phi}}{\sqrt{1-\sin^2\phi\sin^2\theta}}$	$\frac{\cos\theta\sin\phi\hat{\theta}+\cos\phi\hat{\phi}}{\sqrt{1-\sin^2\phi\sin^2\theta}}$
i_y	Hu	$\frac{\cos\theta\sin\phi\hat{\theta}+(\cos\phi+\sin\theta)\hat{\phi}}{1+\cos\phi\sin\theta}$	$\frac{(\cos\phi+\sin\theta)\hat{\theta}-\cos\theta\sin\phi\hat{\phi}}{1+\cos\phi\sin\theta}$
i_z	Hu	$\frac{(\cos\phi+\sin\theta)\hat{\theta}-\cos\theta\sin\phi\hat{\phi}}{1+\cos\phi\sin\theta}$	$\frac{\cos\theta\sin\phi\hat{\theta}+(\cos\phi+\sin\theta)\hat{\phi}}{1+\cos\phi\sin\theta}$

For dual polarization applications, two crossed dipoles, electric and/or magnetic, will produce electric fields orthogonal only in the principal planes (*E*- and *H*-planes). On other hand, the orthogonality of the electric fields of a parallel combination of electric and magnetic dipoles depends on their orientation. As shown in Table III, if those parallel dipoles are vertically oriented along the z-axis, their electric fields (E_θ, E_ϕ) are orthogonal in all directions, but if horizontally oriented along the y-axis, their fields are only orthogonal in the principal planes. Another reference used in this work is a Huygens source (Hu) polarized in the y- or z-direction. Co- and cross-polarization unit vectors of far-field radiation patterns of those

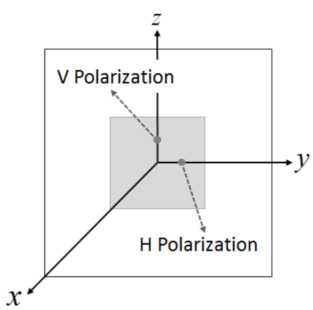

Fig. 3: Dual-polarized antenna element in the y-z plane.

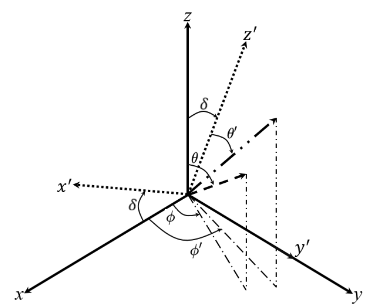

Fig. 4: Spherical coordinate system for a reference and tilted system shown in black solid and dotted, respectively.

sources are summarized in Table III. Following Eq. (9), the dot product of the co- and cross-unit vectors of the two crossed Huygens sources is zero in all directions as follows

$$\hat{u}(i_y)\cdot\hat{u}(i_z)=0 \tag{22}$$

Eq. (22) shows that there is perfect orthogonality between co-polarized radiated fields produced by two orthogonal ports of an dual-polarized antenna. In polarimetric weather radar applications, the H- and V-polarization ports could generate co-polarization components directed in the ϕ and θ direction, respectively, with very low cross-polarization by using electric and magnetic current sources vertically polarized in the z-axis.

From Table III, it can be seen that the proposed polarization basis of orthogonal Huygens sources has rotational symmetry along the antenna's broadside, that is, the electric fields radiated by these sources are orthogonal in all directions. As a result of the rotational symmetry, the mismatching of the H and V copolar radiation patterns will be mitigated. Since the definition of cross-polarization depends on which source is used as the reference, and also on how that source is oriented, various proposed definitions can be used for an antenna lying in the y-z plane, as sketched in Fig. 3.

An additional consideration that must be taken into account is the effect of mechanical elevation tilt on co- and cross-polarization definitions. A tilted system creates an error related to the misprojection of the co- and cross-polar fields onto the local horizontal and vertical directions. Consider a cartesian coordinate system xyz, where the y-z plane is perpendicular to the earth's surface. This system is referred to as the reference coordinate. By rotating the reference coordinate system about the y-axis by some angle δ, a new coordinate system $x'y'z'$ is obtained which is referred to as the primed coordinate. The antenna aperture is positioned at the origin of the coordinate system and placed in the y'-z' plane with its broadside oriented along the positive x'-axis as illustrated in Fig. 4. By using the Euler rotation angle, the unit components $(\hat{\theta},\hat{\phi})$, polar angle (θ), and azimuthal angle (ϕ) can be transferred from one coordinate system to another. The relation between unprimed and primed coordinates can be represented by [13], [14].

$$\begin{bmatrix}\hat{r}'\\ \hat{\theta}'\\ \hat{\phi}'\end{bmatrix}=\begin{bmatrix}\sin\theta'\cos\phi' & \sin\theta'\sin\phi' & \cos\theta'\\ \cos\theta'\cos\phi' & \cos\theta'\sin\phi' & -\sin\theta'\\ -\sin\phi' & \cos\phi' & 0\end{bmatrix}\begin{bmatrix}\cos\delta & 0 & \sin\delta\\ 0 & 1 & 0\\ -\sin\delta & 0 & \cos\delta\end{bmatrix}$$
$$\begin{bmatrix}\sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi\\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi\\ \cos\theta & -\sin\theta & 0\end{bmatrix}\begin{bmatrix}\hat{r}\\ \hat{\theta}\\ \hat{\phi}\end{bmatrix} \tag{23}$$

and

$$\begin{bmatrix}\sin\theta'\cos\phi'\\ \sin\theta'\sin\phi'\\ \cos\theta'\end{bmatrix}=\begin{bmatrix}\cos\delta & 0 & \sin\delta\\ 0 & 1 & 0\\ -\sin\delta & 0 & \cos\delta\end{bmatrix}\begin{bmatrix}\sin\theta\cos\phi\\ \sin\theta\sin\phi\\ \cos\theta\end{bmatrix} \tag{24}$$

After algebraic simplification, the primed unit vectors can be represented in terms of the unprimed unit vectors of the reference coordinate system as follows:

$$\hat{\phi}'=\frac{\sin\phi\sin\delta\hat{\theta}+(\sin\theta\cos\delta+\cos\theta\cos\phi\sin\delta)\hat{\phi}}{\sqrt{1-(\cos\theta\cos\delta-\sin\theta\cos\phi\sin\delta)^2}} \tag{25}$$

$$\hat{\theta}'=\frac{-(\sin\theta\cos\delta+\cos\theta\cos\phi\sin\delta)\hat{\theta}+\sin\phi\sin\delta\hat{\phi}}{\sqrt{1-(\cos\theta\cos\delta-\sin\theta\cos\phi\sin\delta)^2}} \tag{26}$$

This simplifies to

$$\hat{\phi}'=\sin\chi\ \hat{\theta}+\cos\chi\ \hat{\phi}\quad\text{or}\quad\hat{\phi}=\sin\chi\ \hat{\theta}'+\cos\chi\ \hat{\phi}' \tag{27}$$

$$\hat{\theta}'=-\cos\chi\ \hat{\theta}+\sin\chi\ \hat{\phi}\quad\text{or}\quad\hat{\theta}=-\cos\chi\ \hat{\theta}'+\sin\chi\ \hat{\phi}' \tag{28}$$

where

$$\cos\chi=\frac{\sin\theta\cos\delta+\cos\theta\cos\phi\sin\delta}{\sqrt{1-(\cos\theta\cos\delta-\sin\theta\cos\phi\sin\delta)^2}} \tag{29}$$

Assuming the cases listed in Table III, all components are represented in terms of primed unit vectors and angles $(\hat{\theta}',\hat{\phi}',\theta',\phi')$. Now, both the primed vector components and the primed direction parameters must be transferred to the unprimed reference coordinate system using Eqs. (27-29).

V. SIMULATION AND MEASUREMENT RESULTS

As was discussed earlier in this paper, the cross-polarization level using different definitions depends on the source current polarization type and its orientation. Wire and printed half-wave dipoles, a rectangular microstrip patch, a pyramidal horn, and an open-ended rectangular waveguide, designed using the commercial software HFSS [15], were used to illustrate Ludwig's definitions. Measurements were conducted to calculate the co- and cross-polarization components in the principal and diagonal planes. Numerical simulations and measured radiation patterns were obtained at an operating frequency of 3 GHz for all of these antennas. All measurements were performed in the electromagnetic anechoic chamber (EMAC) facility at the Radar Innovations Laboratory (RIL). Since Ludwig's 2nd and 3rd definitions predict the same radiation patterns (co- and cross-polarization) in the principal planes, a comparison between radiation patterns of Ludwig's 2nd and

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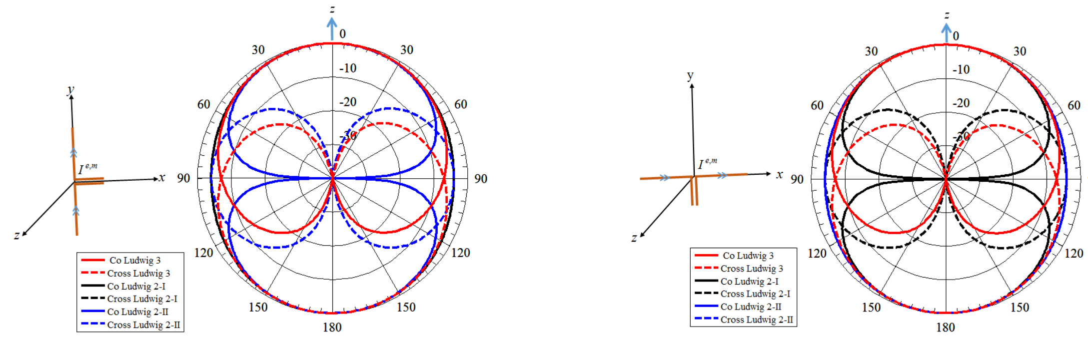
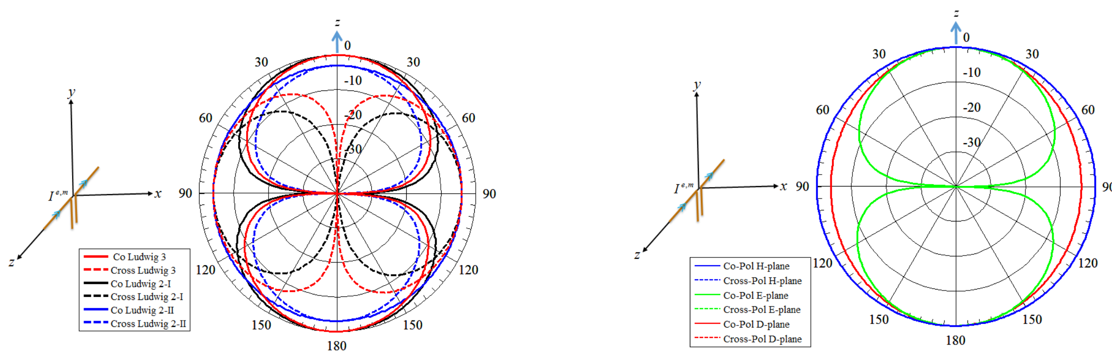


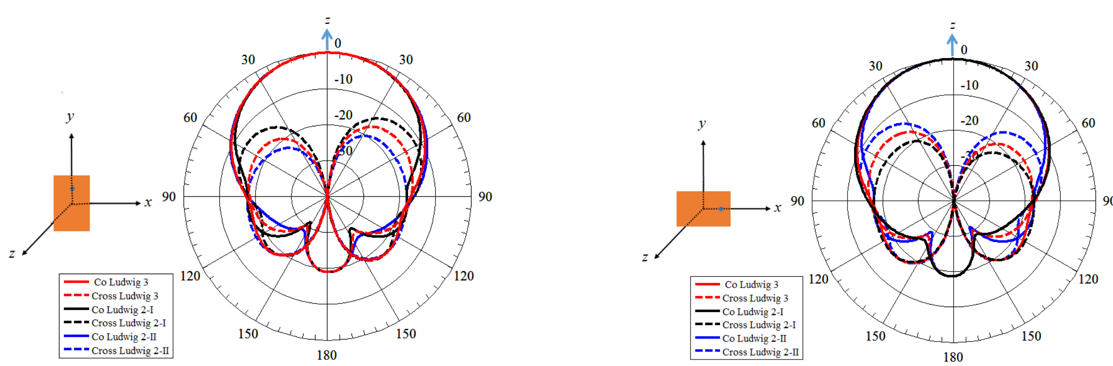
(a) (b)

Fig. 5: Numerical simulations of co- and cross-polarization radiation patterns according to Ludwig's definitions in the *D*-plane of a $\lambda/2$ electric dipole antenna polarized in (a) the y-direction and (b) the x-direction.



(a) (b)

Fig. 6: Numerical simulations of co- and cross-polarization radiation patterns in the *D*-plane of a $\lambda/2$ electric dipole antenna polarized in the z-direction according to (a) Ludwig's definitions (improper definition) and (b) proper definition (E_θ and E_ϕ).



(a) (b)

Fig. 7: Numerical simulations of co- and cross-polarization radiation patterns according to Ludwig definitions in the *D*-plane of a rectangular microstrip patch antenna polarized in (a) the y-direction and (b) the x-direction.

3rd definitions was only conducted in a $45°$ skewed plane. In addition, a Matlab algorithm was used to calculate the simulated and measured normalized co- and cross-polarization patterns in the *D*-plane. For all cases considered in this work, E_θ and E_ϕ components (magnitude and phase) along with equations presented in the paper, were used to calculate co- and cross-polarization radiation patterns.

For its simplicity, a conventional half-wave electric dipole antenna is considered first in this work. For an electric dipole, the co-polarization component is placed in any plane containing the dipole, while the cross-polarization component is placed in any plane orthogonal to the dipole axis. The cross-polarization component of an ideal half-wave dipole is zero. The co-polarization component of its electric field varies as the sine of the angle from the dipole axis, while it is constant in any plane orthogonal to the dipole axis. These characteristics

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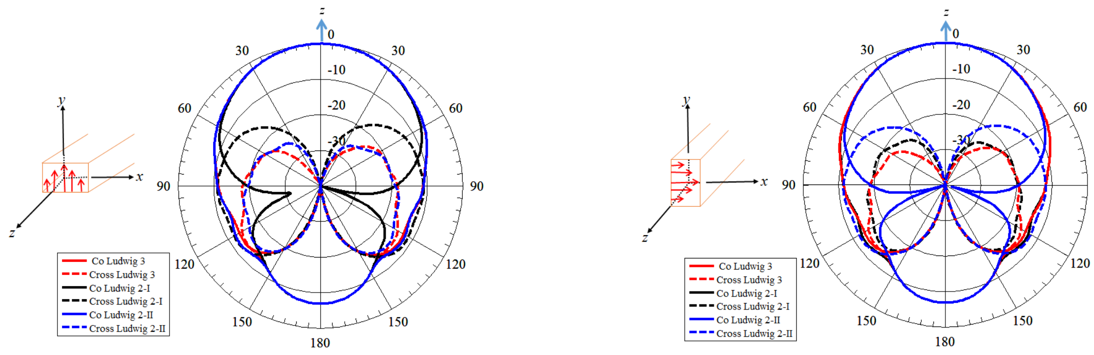

Fig. 8: Numerical simulations of co- and cross-polarization radiation patterns according to Ludwig's definitions in the *D*-plane of an open-ended rectangular waveguide antenna polarized in (a) the y-direction and (b) the x-direction.

are the same for any dipole aligned with the x-, y- or z-axis.

A half-wavelength wire dipole antenna, oriented horizontally along the y- and x-axis, is presented in Fig. 5 (a-b). In this case, the radiated electric field of the dipole, in the far-field region, has both $\hat{\theta}$ and $\hat{\phi}$ components, due to the coordinate system selected (spherical coordinate system). The co- and cross-polarization components of the dipole, polarized in the x- or y-axis, are related to the $\hat{\theta}$ and $\hat{\phi}$ components of the electric fields, according to Ludwig's definitions. The results show a considerable cross-polarization off the principal planes using improper Ludwig equations. According to Ludwig 2-I, a y-polarized dipole has a radiation pattern with very low cross-polarization, ideally zero, in the *D*-plane, as shown in Fig. 5-a. However, if the same dipole is polarized in the x-direction, Ludwig 2-II predicts a very low cross-polarization radiation pattern in the *D*-plane, as shown in Fig. 5-b. The results shown in Table I are consistent with these shown in Fig. 5 (a-b). However, applying Ludwig 2-I and Ludwig 2-II definitions to the x- and y-polarized dipoles, respectively, produces significant cross-polarization degradation in the *D*-plane. It is shown that the cross-polarization level is about -10 dB at $\theta = 45°$ by using Ludwig 2-I with the x-polarized dipole or Ludwig 2-II with the y-polarized dipole. At the same angle, Ludwig 3 shows a cross-polarization level with -15 dB for both orientations. This significant error in the cross-polarization levels is due to the improper use of the definition.

Using the improper definition negatively impacts not only the cross-polarization component, but also the co-polarization component off the principal planes. From Fig. 5, it is apparent that using the improper definition generates a null at $\theta = 180°$ in the co-polarization component while using Ludwig 3 for an electric dipole polarized either in the x- or y-axis. On the other hand, Ludwig 2-II with a y-polarized dipole and Ludwig 2-I with a x-polarized dipole generates a null in the co-polarization component at $\theta = 90°$. Similar behavior can be noticed for the z-polarized dipole. All of these differences can be avoided by using the proper and most meaningful definition.

If the same electric dipole is oriented vertically in the z-axis, its co-polarization component will be θ-directed, while the cross-polarization component with very small value will be ϕ-directed. In this orientation, Ludwig's equations predict inaccurate cross-polarization levels off the symmetry planes as shown in Fig. 6-a. The spherical coordinate bases E_θ and E_ϕ could be used to represent the co- and cross-polarization components, respectively, as shown in Fig. 6-b, where the cross-polarization levels in all the planes are less than -40 dB, ideally zero. This definition is consistent with results shown in Table III. Similar results will be seen if a magnetic dipole is vertically polarized along the z-axis. The only difference is that the co- and cross-polarization components will be ϕ- and θ-directed, respectively. According to polarization characterizations of electric and magnetic dipoles that are polarized vertically, a parallel combination of both electric and magnetic dipoles, polarized vertically, will be an ideal candidate for polarimetric weather radar applications that require polarimetric radars transmit and receive both horizontal and vertical polarizations with very low cross-polarization levels.

The second radiating type used in this work is a rectangular microstrip patch antenna excited in the TM01 mode using a coaxial feed. The length and width of the patch are $L = 31.7$ mm and $W = 39.5$ mm. The microstrip patch antenna was printed on one side of the Rogers RT5880 substrate with a dielectric constant $\epsilon = 2.2$, thickness $t = 1.57$ mm, and a loss tangent of $\tan\delta = 0.0009$. On the other side of the substrate, the ground was printed with size of 10 cm x 10 cm.

Use of equivalent magnetic currents around the patch perimeter reduces the radiation pattern calculation to equivalent slots [7-8]. These slots are considered as magnetic dipoles (equivalent magnetic currents). The equivalent magnetic currents along the radiating edges essentially produce the electromagnetic radiation. However, the equivalent magnetic currents along the resonant length sides (non-radiating edges) weakly radiate (theoretically zero radiation) in the principal planes. In the *H*-plane, because the magnetic current densities on each slot are of the same magnitude but of opposite direction, the fields radiated by these two slots cancel each other. Also both slots on opposite walls are $180°$ out of phase, thus the corresponding radiations cancel each other in the *E*-plane. However, these two nonradiating slots degrade the cross-polarization level away from the principal planes. The radiation intensity of the two nonradiating slots is lower than what is produced by the two radiating slots.

For a linearly polarized patch antenna, if the radiating edges are located along the y-axis, the slot will be considered as a magnetic current polarized in the y-direction. However, a magnetic current will be polarized in the x-direction if the radiating edges are located along the x-axis. Since the patterns are the same in the principal planes, Ludwig's 2nd and 3rd definitions are compared in a $45°$ skew plane.

Numerical results of co- and cross-polarization radiation patterns of a rectangular patch polarized in the y- and x-direction are shown in Figs. 7-a and 7-b, respectively. It is apparent from Fig. 7-a that the y-polarized patch gives a lower cross-polarization level, according to Ludwig 2-II, compared to the other two definitions. According to Ludwig 2-I, the patch polarized in the x-direction, as shown in Fig. 7-b, has a better cross-polarization level compared to that of other definitions. These results are in good agreement with those shown in Table I. There is an 8 dB difference between the cross-polarization levels calculated by Ludwig 2-I and Ludwig 2-II at $\theta = 45°$, and about 4 dB compared to Ludwig 3.

In reality, a linearly polarized patch antenna cannot be considered the same as a pair of perfect magnetic dipoles placed at the radiating edges. It provides only an approximation using the field equivalence principle. In addition, non-radiating slots do radiate away from the principle *E*- and *H*-planes, with weak field intensity everywhere compared to the fields produced by radiating slots. Therefore, the patch antenna can be represented by two magnetic dipoles. However, this antenna with zero equivalent electric current, does not satisfy Huygens source conditions, so using Ludwig 3 will degrade the cross-polarization level.

Another element used is the open-ended rectangular waveguide (OEWG). This antenna is excited with a dominant TE10 mode, and its aperture fields have a cosine taper in the *E*-plane and are uniform in the *H*-plane. This radiating element is the simplest aperture that can be used in array antennas. The standard WR-284 rectangular waveguide was used with a cross section of 72.136 mm x 34.036 mm.

Equivalent magnetic and electric currents are polarized in the x- and y-direction, respectively, when the electric field of the TE10 mode over the antenna aperture is polarized in the y-direction, as shown in Fig. 8-a. On the other hand, if the aperture electric field is polarized in the x-direction, as illustrated in Fig. 8-b, the same currents are obtained after a counterclockwise rotation of $90°$ about the origin. These two equivalent currents are related to each other by the wave impedance η_{TE} at the waveguide aperture. This approaches the characteristic impedance of the free space (377 ohms) as the operating frequency increases above the cut-off frequency. As shown in Fig. 8 (a-b), the Ludwig 3 definition produces lower cross-polarization levels for both orientations. The observed values are not as small as would be expected using Ludwig 3, which should ideally produce zero cross-polarization.

In addition, one of the other definitions closely matches results obtained by the Ludwig 3 definition as shown in Fig. 8 (a-b). The reason is that the OEWG antenna cannot be approximated as a perfect Huygens source. From the *field equivalence principle*, equivalent orthogonal electric and magnetic currents over the OEWG aperture are related by a constant that is not equal to the characteristic impedance of free space. The wave impedance for this mode at the waveguide aperture is greater than the characteristic impedance of free space. Therefore, the equivalent magnetic current will be greater than the equivalent electric current. This case is approximately close to a x-polarized magnetic current source for Fig. 8-a, and a y-polarized magnetic current source for Fig. 8-b.

Fig. 9 illustrates the co- and cross-polarization patterns in the *D*-plane of a printed dipole mounted on a ground plane polarized in the y- and x-direction. It is demonstrated that the printed dipole polarized in the y-direction, as shown in Fig. 9-a, has a lower cross-polarization level, according to Ludwig 2-I. However, radiation patterns of the x-polarized printed dipole have a lower cross-polarization level according to the Ludwig 2-II definition, as shown in Fig. 9-b, compared to other definitions. This is also agree with the earlier analysis summarized in Table I. Ludwig 3 produces about 5 dB of degradation in the cross-polarization level compared to when the proper definition at $\theta = 45°$ is used. On the other hand, the cross-polarization levels using Ludwig 2-II in Fig. 9-a, and Ludwig 2-I in Fig. 9-b, are very high compared to that calculated according to Ludwig 2-I and Ludwig 2-II, respectively, in the same figures. This difference is about 10 dB. This inaccuracy in the cross-polarization levels has a large impact on antenna performance in applications requiring very low cross-polarization levels. This is the case in polarimetric weather radars that require less than -40 dB cross-polarizations for $\pm 45°$ scan volume in the *E*-, *H*-, and *D*-planes [16].

To verify the simulated results, E_θ and E_ϕ components of an electric far field radiated by a microstrip patch and a pyramidal horn antenna are measured, both in amplitude and phase, in the *D*-plane, as shown in Figs. 10 and 12. These components, obtained from the far-field chamber measurements, were then used to calculate the co- and cross-polarization components based on Ludwig's definitions. Using measured E_θ and E_ϕ, the calculated co- and cross-polarization components are shown in Figs. 11 and 13 for the rectangular microstrip patch and pyramidal horn antennas, respectively. In Fig. 11 (a-b), according to the direction of patch antenna polarization, cross-polarization levels are different according to the used Ludwig's definition. The same can be observed in Fig. 13 (a-b) from the results using the pyramidal horn antenna.

As explained early in this paper, Ludwig 2-I is defined according to the reference field radiated by a linearly polarized electric dipole along the y-axis. However, the reference field radiated by a linearly polarized electric dipole along the x-axis defines Ludwig 2-II. Ludwig's third definition is defined by the E-field radiated by a Huygens source. With an ideal case, these analytical expressions predict zero cross-polarization. In practice, in addition to intended polarization currents, other currents will contribute to cross-polarization. For simplicity, a Ludwig 2-I is considered as a reference definition and is used with an ideal linearly polarized source such as an electric dipole polarized in the y-direction. This dipole is simulated in HFSS with a simple feed structure (a lumped port feed) to reduce its impact. The simulated cross-polarization values are very low, and they are comparable to analytical values. Several antenna types designed at 3 GHz, including

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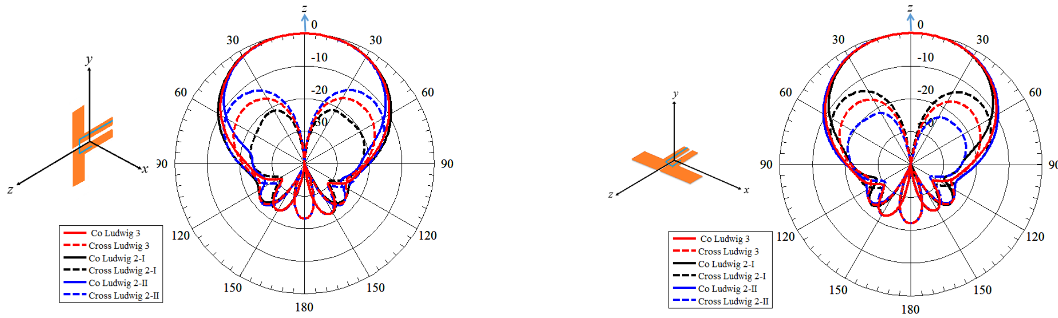
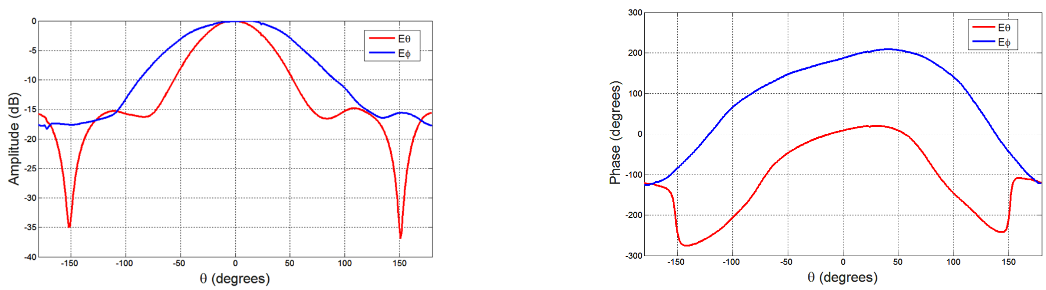


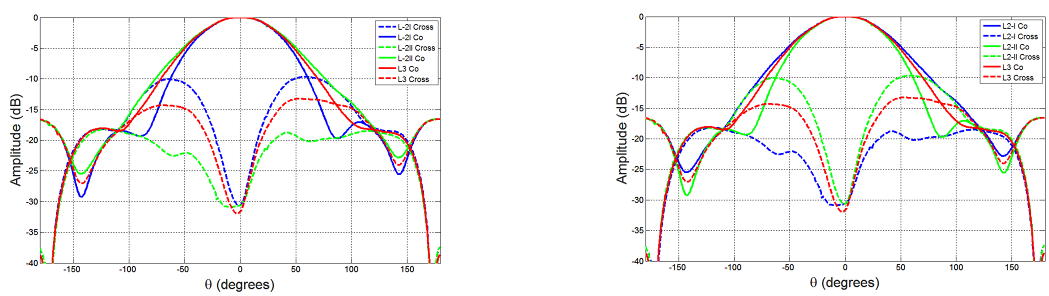
(a) (b)

Fig. 9: Numerical simulations of co- and cross-polarization radiation patterns according to Ludwig's definitions in the D-plane of a printed dipole antenna polarized in (a) the y-direction and (b) the x-direction.



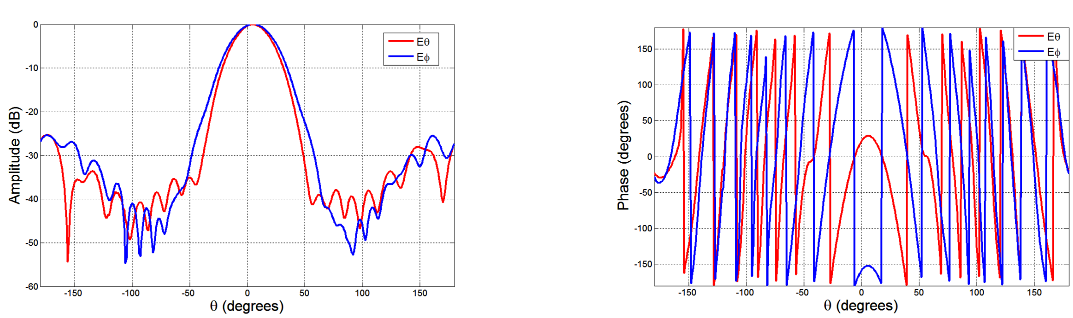
(a) (b)

Fig. 10: Measured E_θ and E_ϕ components in the D-plane of a microstrip patch antenna (a) amplitude (dB) and (b) phase (°).



(a) (b)

Fig. 11: Measured antenna patterns of a microstrip patch antenna using Ludwig's definitions when the current source is polarized in (a) the x-direction and (b) the y-direction.



(a) (b)

Fig. 12: Measured E_θ and E_ϕ components in the D-plane of a pyramidal horn antenna (a) amplitude (dB) and (b) phase (°).

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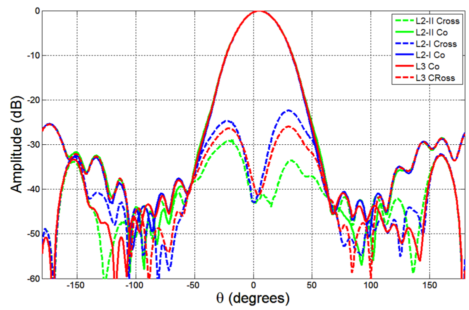
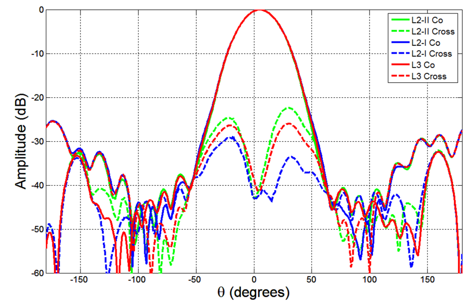


(a)



(b)

Fig. 13: Measured antenna patterns of a pyramidal horn antenna using Ludwig's definitions when the current source is polarized in (a) the x-direction and (b) the y-direction.

wire dipole, printed dipole, narrow-side-fed rectangular patch, wide-side-fed rectangular patch, single-polarized square patch, dual-polarized square patch, and dual-polarized crossed patch were used in this study to demonstrate the errors induced by improper use of Ludwig's definitions. These antennas are numbered from 1 to 7, respectively, as shown in Fig. 14.

The induced error (Δ) by improper use of Ludwig's definitions represents the difference between the cross-polarization levels using the proper definition and other improper definitions. For an ideal linearly y-polarized source, Ludwig 2-I is considered as a reference definition and Ludwig 2-II and Ludwig 3 are improper ones.

For the simple wire dipole, using improper definitions gives about 35 dB and 41 dB errors, respectively, according to Ludwig 3 and Ludwig 2-II as shown in Table IV. In this case, using the right definition is very important. Cross-polarization levels are calculated in the D-plane at $\theta = 45°$. For the printed dipole, there is a 12 dB error, according to the L 3 definition, and 18 dB error, according to the Ludwig 2-II definition. From Table IV, it is apparent that these errors are reduced to about 4 dB and 8 dB, respectively, according to the L 3 and Ludwig 2-II definitions for the patch antennas. Because of the antenna geometry or the feeding structure, the leakage radiation will be produced with large component perpendicular to the intended polarization. This mechanism will reduce the error.

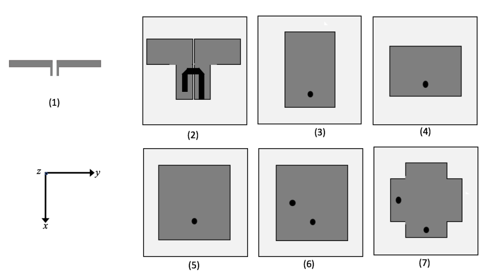
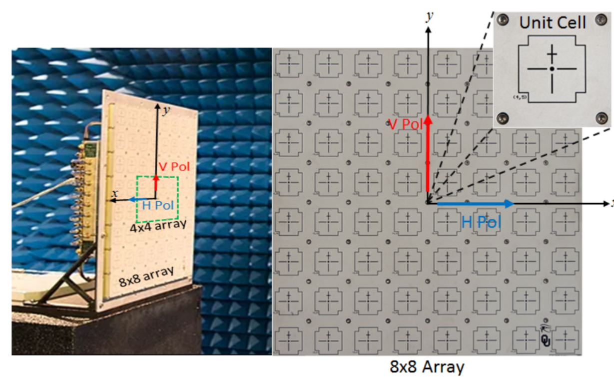


Fig. 14: Geometry of proposed antennas: wire dipole (1), printed dipole (2), narrow-side-fed rectangular patch (3), wide-side-fed rectangular patch (4), single-polarized square patch (5), dual-polarized square patch (6), and dual-polarized crossed-patch antenna (7).

Since the cross-polarization of the antenna element affects the cross-polarization of the antenna array, a unit cell antenna

TABLE IV: Errors in cross-polarization levels caused by improper use of Ludwig's definitions for several antennas numbered from 1 to 7, respectively, as shown in Fig. 14

Error Δ (dB)	An. 1	An. 2	An. 3	An. 4	An. 5	An. 6	An. 7
L 2-II	40.7	18.0	8.6	8.6	8.4	7.6	8.5
L 3	34.7	12.5	2.4	4.3	4.2	4.4	5.2



(a) (b)

Fig. 15: Near-field chamber setup for electronic scanned radiation patterns of an array 4x4 elements embedded in an array of 8x8 elements at 3 GHz. (b) Top-view of the 8x8 antenna array and unit cell element [16].

element should be designed to attain a minimum level of cross-polarization, not only at broadside, but also within the angular scan range. Typically, active phased array antennas are providing azimuth and elevation electronically scanned ranges from $-45°$ to $+45°$. Within this angular scan range the cross-polarization level should be as low as possible.

To extend the theory to a dual-polarized antenna element and array, a high performance antenna element with dual-polarization, wide scan angle, and low cross-polarization levels for phased array radars, designed for fully digital multifunction phased array radars in [16], is used. This antenna requires very low cross polarization level of 40 dB in the scanning range. This is one of our motivations to revisit the cross-polarization definitions and correctly choose the proper cross-

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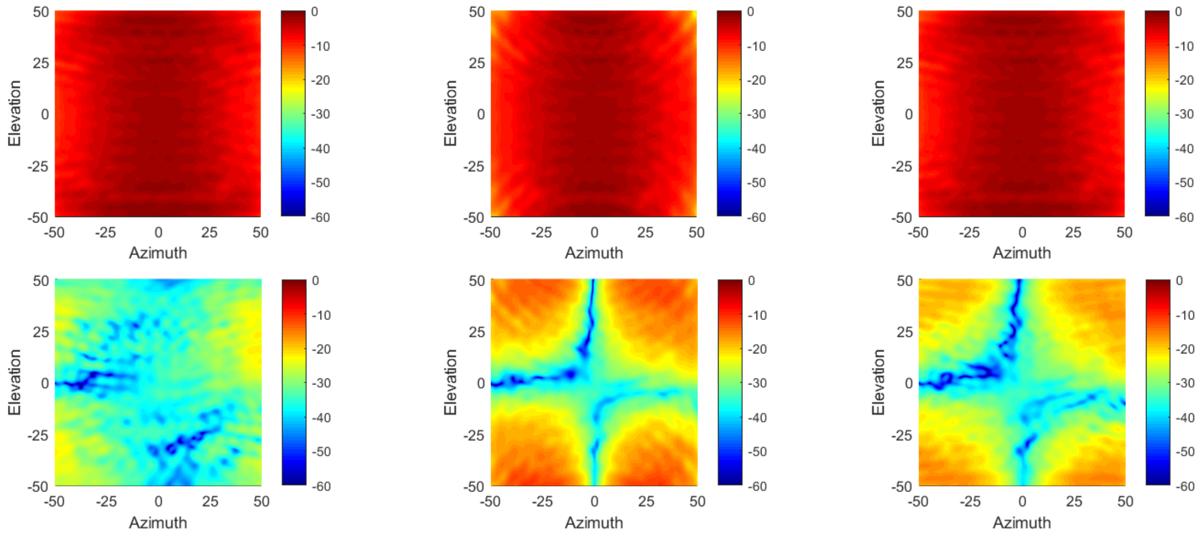
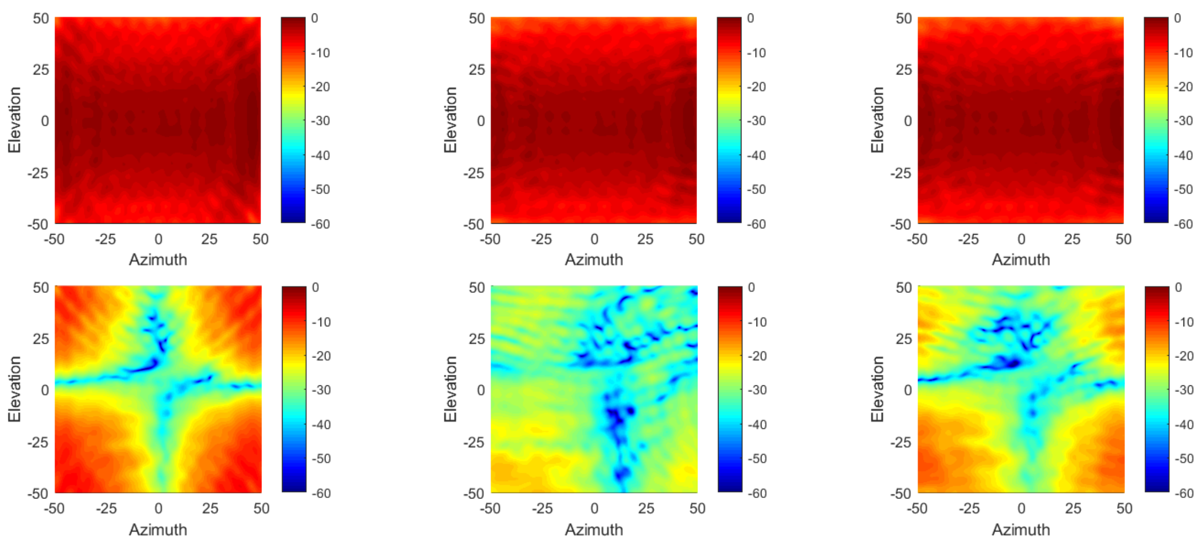


(a) (b) (c)

Fig. 16: Measured radiation patterns of the unit cell antenna at 3 GHz with H-polarization; co- and cross-polarization magnitude in dB according to (a) Ludwig 2-I, (b) Ludwig 2-II (second row), and (c) Ludwig 3.



(a) (b) (c)

Fig. 17: Measured radiation patterns of the unit cell antenna at 3 GHz with V-polarization; co- and cross-polarization magnitude in dB according to (a) Ludwig 2-I, (b) Ludwig 2-II (second row), and (c) Ludwig 3.

polarization definition will be used. The antenna architecture and characteristics, in element or array level, was illustrated in [16]. Near-field chamber setup for electronic scanned radiation patterns of an array 4x4 elements embedded in an array of 8x8 elements at 3 GHz as shown in Fig. 15 (a-b).

Measured radiation patterns at 3.0 GHz for H- and V-polarizations of the unit cell on a $\lambda/2$ ground plane to minimize the possibility of grating lobes were conducted. The co- and cross-polarization radiation patterns of the unit-cell antenna were calculated using all Ludwig definitions for both polarizations. Fig. 16 shows that for the H-polarization the Ludwig 2-I gives the lower cross polarization compared to others. For V-polarization, lowest cross-polarization level is obtained by using Ludwig 2-II as shown in Fig. 17. The induced error by improper definitions were calculated and presented in Table V after normalize all cross-polarization values to the lowest value obtained by suing the proper definition. From the table, the difference between them looks to be of the order of 7 dB or 12 dB. By further analyzing the cross-polarization radiation patterns of the dual-polarized element antenna, it is obvious that the *E*- and *H*-planes would not show the maximum values of the cross-polarization. However, in the diagonal plane, the differences in cross-polarization levels clearly identifiable by the plots in Figs. 16 and 17.

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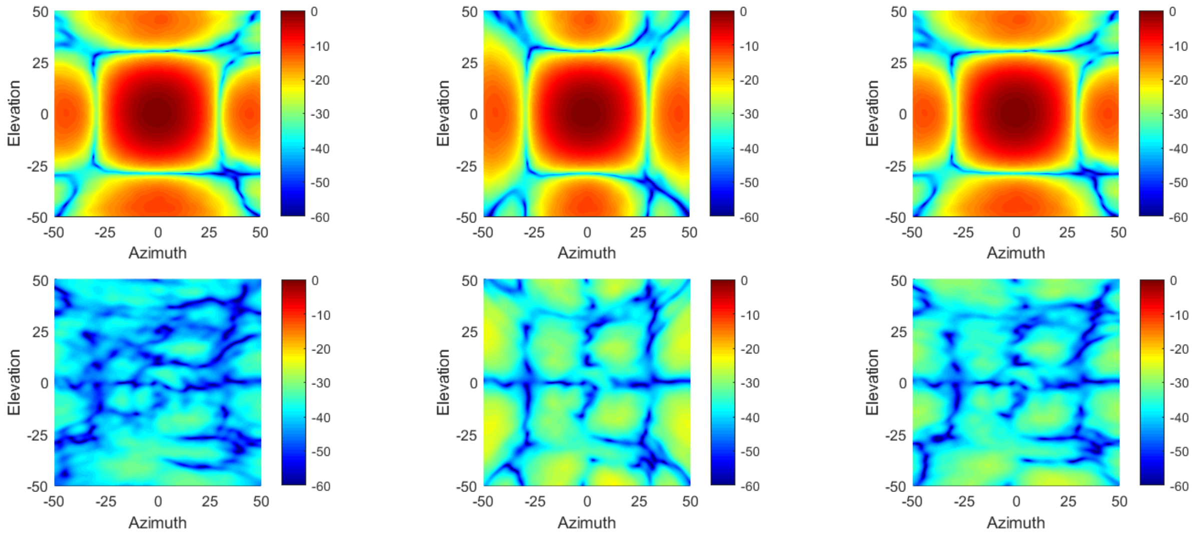
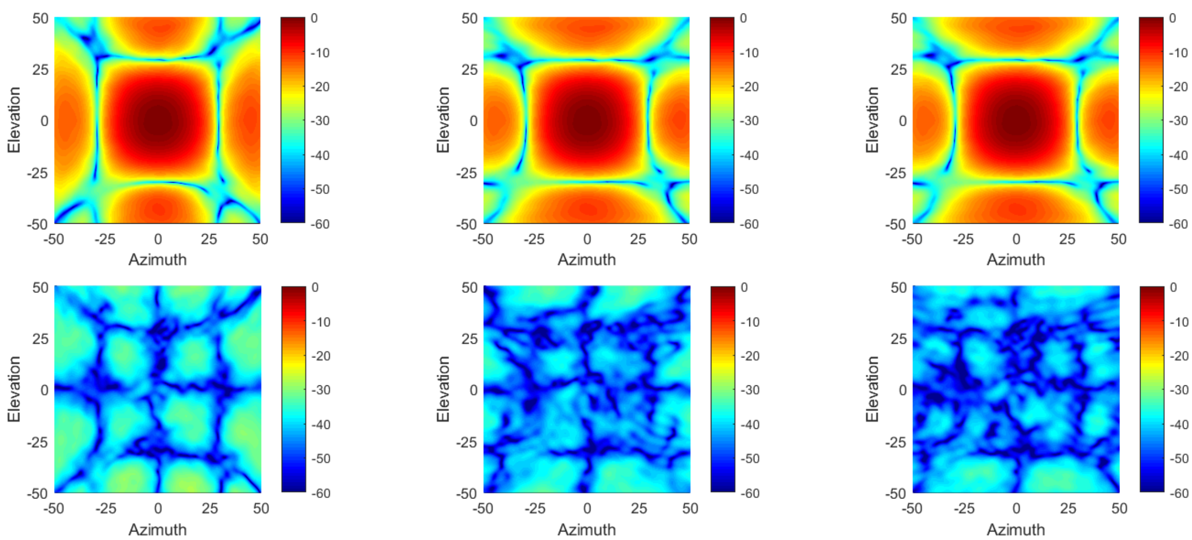


(a) (b) (c)

Fig. 18: Measured polarimetric 4x4 planar phased array radiation patterns with H-polarization at broadside; co- and cross-polarization magnitude in dB according to (a) Ludwig 2-I, (b) Ludwig 2-II (second row), and (c) Ludwig 3.



(a) (b) (c)

Fig. 19: Measured polarimetric 4x4 planar phased array radiation patterns with V-polarization at broadside; co- and cross-polarization magnitude in dB according to (a) Ludwig 2-I, (b) Ludwig 2-II (second row), and (c) Ludwig 3.

Similarly, the applications are extended to the active phased array antenna with V- and H-polarizations. The used array is 4X4 elements embedded in an array of 8x8 elements with a uniform distribution and constant phase difference between the elements. Planar near-field antenna measurements have been performed in the NF chamber. The same conclusion can be noticed for the broadside angle in the diagonal plane.

Figs. 18 and 19 show the co- and cross-polarization radiation patterns of the broadside beam of H- and V-polarizations. It is apparent that the lowest cross-polarization levels can be obtained by Ludwig 2-I for the H-polarization and by Ludwig 2-II for the V-polarization. The errors between the cross-polarization levels obtained by different definitions look to be of the order of 6 dB and 5 dB for H-polarization and V-polarization, respectively, as shown in Table V. For the scanned beam from the broadside direction, the cross-polarization will be higher and the induced errors between the definitions will be large. Since the used dual-polarized antenna does not have a perfect symmetry property with respect to the two feeding ports, the radiation patterns of both polarizations will not show the same level of the cross-polarization. In order to meet the requirements mentioned up, a square antenna design with a complex multi-layer structure and complex feeding network is required [16]. This complexity in the feeding and structure

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TABLE V: Errors induced by using improper definitions of the dual-polarized element antenna and active dual-polarized scanned array antenna

Pol	Ludwig defi.	Antenna element	Array Broadside scann
H	L 2-I	0	0
	L 2-II	11	8
	L 3	7	5
V	L 2-I	12	6
	L 2-II	0	0
	L 3	7	4

causes undesired radiation which will degrade the cross-polarization levels. In other word, instead of having a prefect magnetic current linearly polarized in one direction, another undesired magnetic current polarized in the other direction. This undesired current will degrade the used definition derived based on the perfect linearly polarized currents.

VI. Conclusions

Ludwig's definitions have been intensively discussed and clarified. These definitions were derived assuming a certain radiating source polarized in a specified direction (in the x-y plane) as a reference to define co- and cross-polarization components. However, in real applications, antennas can be located in the x-y, x-z, or y-z plane. In this contribution, the co- and cross-polarization definitions have been generalized by using different antenna sources located in the y-z or x-z plane to complement Ludwig's definitions.

In addition, this work illustrates the degradation that can occur in the cross-polarization level if the improper definition is used. This degradation can be significantly attributed to the field projection, which is mainly dependent upon the definition used. Based on the current definitions, Ludwig 2 assumes perfectly linear-polarized radiating sources, oriented in the x- or y-direction with zero component in the orthogonal direction. On the other hand, Ludwig 3 assumes a radiating source that satisfies Huygens conditions. In practice, these assumptions are difficult to satisfy. Most practical radiating elements have another coupled current in the orthogonal direction which does not satisfy the Huygens conditions. Therefore, to demonstrate the new extended formulation presented in this paper, extensive tradeoffs using different radiating sources in different planes have been used. For linearly-polarized practical antennas, errors will be introduced in cross-polarization calculation because of deviations from the ideal antennas used to derive Ludwig definitions, in contrast with real antennas. Using the proper definition will produce cross-polarization with lower values only if the antennas are close to ideal conditions.

For an antenna oriented in planes other than the x-y plane, the proper definition of the co- and cross-polarization needs to be defined based on the reference type. For example, the spherical coordinate bases (E_θ and E_ϕ) are highly recommended to define the co- and cross-polarization components of a electric or magnetic dipole polarized vertically in the z-axis.

From an educational point of view, this paper revisits Ludwig's formulations of the co- and cross-polarization. The described work provides supplementary material for teaching graduate level antenna theory and radar, and thus serves as a good reference for faculty members, antenna engineers, and graduate students. Ludwig's definitions are primarily defined for an infinitesimal electric dipole, infinitesimal magnetic dipole, or Huygens antenna. Each of these definitions is applied to a corresponding optimal source, based on its type and orientation.

VII. Appendix

The corresponding E-field components that are due to J_s and/or M_s, polarized in the x- and/or y-direction, can be calculated using [11-12]

$$E_\theta \simeq -\frac{j\beta\ e^{-j\beta r}}{4\pi r}\ (L_\phi + \eta N_\theta) \tag{30}$$

$$E_\phi \simeq +\frac{j\beta\ e^{-j\beta r}}{4\pi r}\ (L_\theta - \eta N_\phi) \tag{31}$$

where N_θ, N_ϕ, L_θ, and L_ϕ are given in [11-12]

By specifying the equivalent current density, either J_s of a wire antenna, or J_s and M_s over the close surface of an aperture antenna, the radiated E-field components can be determined. The equivalent current densities J_s and M_s over S of the aperture are calculated using [11-12]

$$\mathrm{J}_s = \hat{\mathrm{n}} \times \mathrm{H}_a \qquad , \qquad \mathrm{M}_s = -\hat{\mathrm{n}} \times \mathrm{E}_a \tag{32}$$

where $\hat{\mathrm{n}}$ = unit vector normal to the surface S, E_a and H_a are total electric and magnetic fields over the surface S.

For electric and magnetic currents I^e and I^m, (30-32) reduce to line integrals. The far-zone components of the electric field of an electric current, oriented along the x-axis, are given by

$$E_\theta \simeq -\frac{j\eta\beta\ e^{-j\beta r}}{4\pi r}\cos\theta\cos\phi\int_C I_x^e\ e^{j\beta r'\cos\psi}dl' \tag{33}$$

$$E_\phi \simeq +\frac{j\eta\beta\ e^{-j\beta r}}{4\pi r}\sin\phi\int_C I_x^e\ e^{j\beta r'\cos\psi}dl' \tag{34}$$

and when the same electric current is oriented in the y-axis, the far-zone components of the electric field are given by

$$E_\theta \simeq -\frac{j\eta\beta\ e^{-j\beta r}}{4\pi r}\cos\theta\sin\phi\int_C I_y^e\ e^{j\beta r'\cos\psi}dl' \tag{35}$$

$$E_\phi \simeq -\frac{j\eta\beta\ e^{-j\beta r}}{4\pi r}\cos\phi\int_C I_y^e\ e^{j\beta r'\cos\psi}dl' \tag{36}$$

For a magnetic current I^m oriented along the x-axis, the far-zone components of the electric field are given by

$$E_\theta \simeq +\frac{j\beta\ e^{-j\beta r}}{4\pi r}\sin\phi\int_C I_x^m\ e^{j\beta r'\cos\psi}dl' \tag{37}$$

$$E_\phi \simeq +\frac{j\beta\ e^{-j\beta r}}{4\pi r}\cos\theta\cos\phi\int_C I_x^m\ e^{j\beta r'\cos\psi}dl' \tag{38}$$

and when this magnetic current is oriented in the y-axis, the far-zone components of the electric field are

$$E_\theta \simeq -\frac{j\beta\ e^{-j\beta r}}{4\pi r}\cos\phi\int_C I_y^m e^{j\beta r'\cos\psi}dl' \tag{39}$$

$$E_\phi \simeq +\frac{j\beta\ e^{-j\beta r}}{4\pi r}\cos\theta\sin\phi\int_C I_y^m e^{j\beta r'\cos\psi}dl' \tag{40}$$

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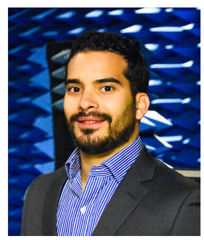

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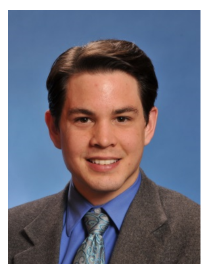

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