A New Analytical Model Based on Diffraction Theory for Predicting Cross-polar Patterns of Antenna Elements in a Finite Phased Array

Javier A. Ortiz, *Student Member, IEEE,* Nafati Aboserwal, *Member, IEEE* and
Jorge L. Salazar, *Senior Member, IEEE*

***Abstract*—This paper presents a mathematical model based on diffraction theory to predict the co- and cross-polar patterns of phased array antenna elements. The technique provided is used on the theoretical model of a monopole antenna and is supported by simulated and measured values of the radiating element in different locations of a ground plane. An extension of diffraction theory based on the equivalent current model is used to estimate the cross-polarization patterns of the monopole. This proposed method proves as an accurate representation of the radiating characteristics of the isolated and embedded element patterns in a finite array antenna. Good agreements were found between results of the proposed method, numerical simulations, and measurements.**

***Keywords*—Embedded element pattern, diffraction, equivalent currents, cross-polarization.**

I. Introduction

In recent years, antenna performance requirements in cross-polarization levels have pushed boundaries in design. One good example is the design of radiating elements for the use of weather radar phased array technology. One such application is simultaneous transmit and simultaneous receive (STSR) polarization mode, where the cross-pol level requirements are down to -40 dB for scanning in principal planes and -35 dB for scanned patterns in off-principal planes [1], [2]. The design of arrays for weather radar applications require large panels that are produced in tiles or subarrays to comply with fabrication and mechanical limitations. This introduces gap discontinuities between subarrays when mounted in the front panel of the system.

Discontinuities in the conductive plane of antennas produce diffracted field levels that can reach about -30 dB and affect the performance of elements the closer they are to the discontinuities or edges of finite arrays [3]. These fields can greatly disrupt the cross-polarization of individual elements and can result in higher levels when scanning an array off of broadside and at off-principal planes. This should be of great importance especially for cases like large fully-digital phased arrays, where individual elements are excited independently to form multiple beams from multiple sections of the array [4]. The effects of the discontinuities could greatly contaminate the cross-polarized fields for individual scanned patterns. A mathematical model is ideal to predict such scanned patterns where an extensive array might require too much resources for numerical simulations.

J. A. Ortiz, N. Aboserwal, and J. L. Salazar are with the School of Electrical and Computer Engineering and the Phased Array Antenna Research & Development (PAARD) team of the Advanced Radar Research Center (ARRC) at The University of Oklahoma, Norman, OK, 73071 USA.

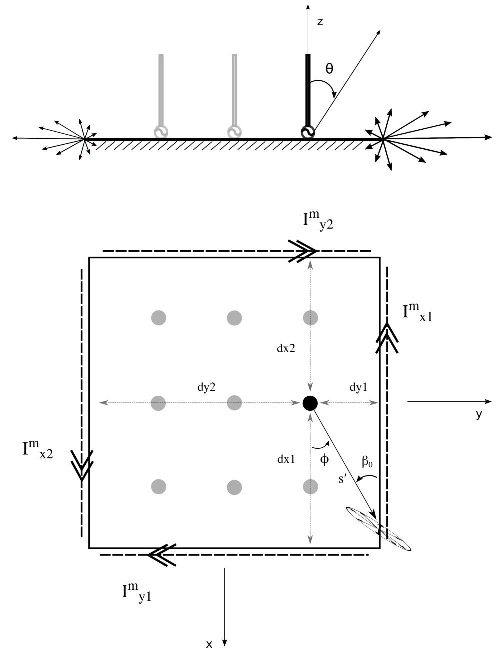

Fig. 1: Illustration of diffracted fields and equivalent currents generated by a monopole antenna of about $\lambda/4$ in length on a finite ground plane.

There has been previous attempts to model the embedded elements of an array and their cross-polar components including the mutual coupling in the presence of edge effects [5]. However, this does not take into account the diffraction effects directly in the pattern. The element patterns can be affected by the presence of mutual coupling as well as edge effects [6]. This work only considers the effect in the mutual coupling parameters and no emphasis done to cross-polar components of the radiation pattern. For monopole antenna arrays the effect of mutual coupling is considered theoretically and in experiments [7]. Yet, there hasn't been a study where the individual embedded elements and the contribution of edge effects to the shape and amplitude of the individual cross-polar patterns based on their location in a finite ground plane.

The calculation of fields present in the radiating element's illuminated and shadow regions, usually referring to the front and backside of the ground plane respectfully, are made possible with the use of the Uniform Theory of Diffraction (UTD) [8]. This theory provides a dyadic diffraction coefficient capable of producing an approximation based on an asymptotic solution of the diffracted fields due to discontinuities in conductive surfaces and wedges. Further implementation of this model is seen for calculating diffraction due to a geometry's finite edge where radiation integrals are necessary to produce the fields at the caustic region by means of equivalent currents [9]. The use of the UTD method and its extension, to what is known as the equivalent current method, is used to evaluate diffraction from ground plane edges in common antenna elements, such as aperture antennas, in both its principal E- and H-planes [10]. This provides a combination of diffraction techniques and radiation integrals in order to produce a solution for the fields produced by wedges of finite lengths.

This manuscript continues work on the known presence of diffracted fields produced by the edges in a finite array and provides a solution to predict not only the co-polar but also the cross-polarized components of such phenomena. Section II, provides the equations that determine both co- and cross-pol patterns of an individual monopole element based on its position with respect to the edges. The theoretical results, simulated, and measured results are discussed in Section III. Finally, a conclusion of this study is shown in Section IV.

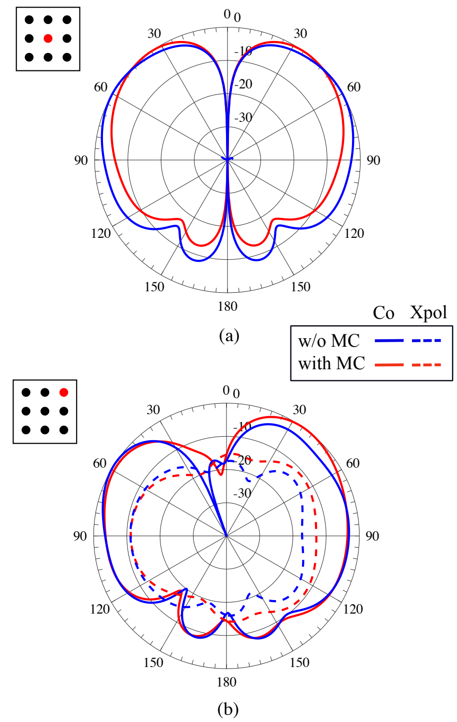

Fig. 2: A comparison of a simulated monopole element placed in the (a) center [0,0] position and (b) corner [1,1] position of a $\lambda/2$ spacing 3x3 configuration with and without neighboring elements for mutual coupling.

II. Proposed Analytical Model

The monopole antenna, as seen in Fig. 1, is a fundamentally basic element that theoretically produces E_θ components along all ϕ angles. It is chosen in this manuscript because its theoretical value for the cross-polarized component (E_ϕ) is zero. Fig. 2a and b show a comparison of simulated monopole antenna patterns in its array environment in two different positions with and without neighboring elements that introduce mutual coupling. This clearly demonstrates that the monopole antenna in the center is purely polarized in θ, even with mutual coupling introduced, while the ϕ components are present solely due to diffracted fields from the edges.

The most straightforward way to predict the radiation pattern of an element with the effects of the diffracted fields from the edges of a conducting surface is by calculating the two-point diffraction with the use of the dyadic coefficients in diffraction theory. The diffraction coefficients are determined by the distance from the source to the point of diffraction, the angle of incidence, and the geometry of the wedge. This method only needs a point of diffraction on each side of the ground plane in a single cut in azimuth (ϕ). For the case of this study the ground plane has four straight edges and assumed to be a strip where the wedge has no angle.

Since, for the case of the monopole antenna, E_ϕ is theoretically zero, the diffracted fields using the two-point diffraction can only be determined for E_θ components at any point Q_D along the edge, shown as $E_\theta^\mathrm{i}(Q_\mathrm{D})$.

The diffracted fields E^D are determined by:

$$\begin{bmatrix} E_\phi^\mathrm{D}(s) \\ \\ E_\theta^\mathrm{D}(s) \end{bmatrix} = -\begin{bmatrix} D_s & 0 \\ 0 & D_h \end{bmatrix} \begin{bmatrix} E_\phi^\mathrm{i}(Q_\mathrm{D}) \\ \\ E_\theta^\mathrm{i}(Q_\mathrm{D}) \end{bmatrix} A(s', s) e^{-jks} \tag{1}$$

where k is the propagation constant, s' the distance from the source to the point of diffraction, D_h and D_s are known as hard and soft diffraction coefficients and $A(s', s)$ is a spatial attenuation parameter [11]. The diffraction coefficient used is determined by the orientation of the incident field upon the wedge. All incident field components from the monopole are perpendicular to the edge, therefore, only hard diffraction coefficient is considered. Considering that E_ϕ components are zero and the grazing angle of the incident field with respect to the surface is considered to be $0°$, the soft diffraction coefficient is omitted and only E_θ^D diffracted fields can be determined. However, due to the diffraction phenomena itself there are diffracted fields that have E_ϕ components in practice, as seen in Fig. 2b.

The previous equations are sufficient for the co-polar components. To obtain the other component contributions coming from currents aligned in the opposite direction and the interaction along all the edge must be considered as a line of current in space. The equivalent current technique can model currents along the (x, y) plane, as shown in Fig. 1, producing both

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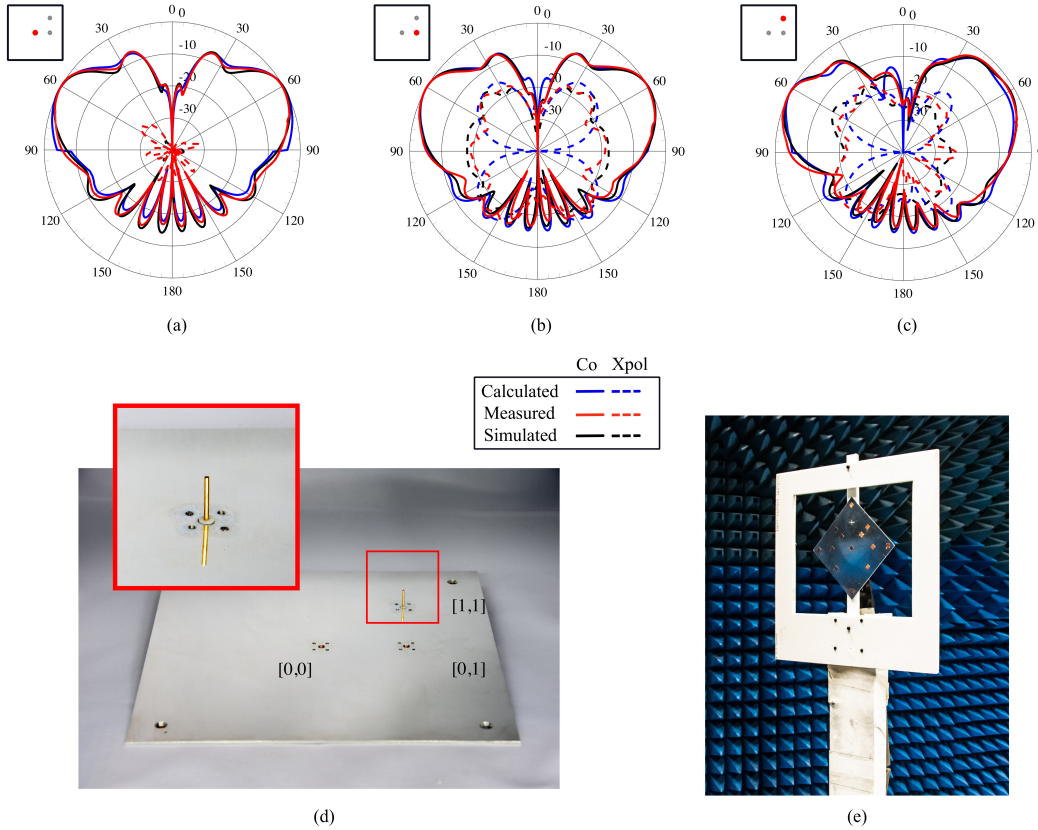

Fig. 3: Comparison of theoretical (proposed method), simulations, and measured results of isolated monopole antenna patterns with the effect of diffracted fields on co- and cross-pol when placed at (a) center [0,0] position, (b) edge [0,1] position, and (c) corner [1,1] position on a 4λ sized ground plane. The (d) relatively thin (1.57 mm) aluminum sheet is (e) mounted on an electromagnetically invisible pedestal for far-field measurements.

far-field components using vector potentials.

A magnetic current I^{m} is used to represent the diffracted fields from such incident E_θ fields [11].

$$I^{\mathrm{m}}_{x,y} = -\eta H^{\mathrm{i}}_{x,y}(Q_D)\sqrt{\frac{8\pi}{k}}D_h e^{-j\frac{\pi}{4}} \tag{2}$$

$H^{\mathrm{i}}_{x,y}$ being the incident magnetic field at any point in x and y. Once the magnetic current is determined for each point along the edge, the radiation integrals using vector potential F for a magnetic current lines are calculated [11].

$$\mathbf{L} = \iint_S \mathbf{I}^{\mathrm{m}}_{x,y} e^{jks'\cos\psi}\,\mathrm{d}s' \tag{3}$$

$$\mathbf{F} = \frac{\epsilon e^{-jkR}}{4\pi R}\mathbf{L} \tag{4}$$

$$\begin{bmatrix} E^{\mathrm{D}}_\phi = j\omega\eta F_\theta \\ E^{\mathrm{D}}_\theta = -j\omega\eta F_\phi \end{bmatrix} \tag{5}$$

Now that both components are determined, they can be added to the total radiated field:

$$\mathbf{E}^{\mathrm{Total}}(\theta,\phi) = \mathbf{E}^{\mathrm{GO}}(\theta,\phi) + \mathbf{E}^{\mathrm{D}}(\theta,\phi) \tag{6}$$

where E^{GO} is the monopole's geometric optics (GO) electric field pattern, which takes into account the reflections of an infinite conductive surface, and E^{D} is the diffracted electric field from the calculated vector potential of the equivalent currents at the edges taking into account both E_θ and E_ϕ components.

Having all mutual coupling parameters (S_{mn}) between the element of interest and its neighboring elements, one can add these diffracted fields to both compontents of the radiation pattern in order to have a more accurate and complete representation of the embedded element pattern.

$$\mathbf{E}^{\mathrm{e}}_m(\theta,\phi) = \left(\mathbf{E}^{\mathrm{isol}}(\theta,\phi) + \mathbf{E}^{\mathrm{D}}_m(\theta,\phi)\right)\left(1 + \sum_n \mathbf{S}_{mn} e^{-jk(r'_n - r'_m)\cdot\hat{r}}\right) \tag{7}$$

This proposed equation for an embedded element pattern (E_m^{e}) placed in any arbitrary position (m) of the conductive surface or ground, now includes the effects of diffracted fields from the edges at each location (E_m^{D}) as well as the mutual coupling parameters of the array to an isolated element pattern, represented by E^{isol}. It also takes into account the cross-polarized component when it is usually neglected.

III. Result Discussion

In theory, one of the sources of cross-polarization contamination is known to be mutual coupling of antenna elements in an array [12]. There is cross-polarization increase whenever an element is in the presence of mutual coupling as does its radiation characteristics in general due to changes in its input impedance. However, Fig. 2 shows that diffraction will be a predominant factor introduced into specific antenna elements, especially when the fields that are being diffracted originate from asymmetrical positioning of the elements.

The monopole cases presented in Fig. 3 show proof that a highly-pure element such as a line of current along z, presented here as a monopole, cross-polarized fields of high levels are introduced entirely out of diffraction from the edges. This can be confirmed by the theoretical model presented in this manuscript where the element at the center, shown in Fig. 3a, has no cross-polarized fields due to cancellation of symmetrical equivalent currents. When the antenna element is moved asymmetrically (off-center), as illustrated in Fig. 3b and c, cross-polarization levels increases significantly due to added E_ϕ components. This is because contributions of diffracted fields from edges that are illuminated asymmetrically do are being amplified by the edges and not cancelling.

When moved to an asymmetrical position such as Fig. 3b, the points of diffraction in currents along y are of the same distance, therefore when looking at the cut in $\phi = 0°$ the co-polarized pattern (E_θ) looks symmetrical, but the cross-polarized fields (E_ϕ) come from asymmetrical sources since the equivalent current from diffraction is stronger along $+x$ removing the ability for these fields from both x currents to counteract each other. The result is a cross-polarized pattern increase from well under -40 dB to greater than -20 dB. The theoretical model satisfactorily predicts this rise in cross-polarized fields, especially around boresight ($\theta = \pm 60°$) where phased array patterns are usually scanned.

IV. Conclusion

For most array antenna elements, mutual coupling is a very critical component in the performance of the array. In some cases like microstrip patches it can greatly affect the cross-polarization performance. Results in this manuscript point to the fact that if it is critical to have low cross-polarization patterns for scanning arrays and there is a presence of periodic conductive surface discontinuities, diffraction can be an even greater limitation towards desired performance. In the case of the monopole shown, mutual coupling is barely a contributor to increases in cross-polarization patterns. The steps shown here provide a more accurate representation of what diffraction will contribute to an antenna element's performance in such environment.

It is proven with this concept by means of calculations, simulations, and measurements that cross-polarized fields are generated significantly by antenna elements positions asymmetrically and close to the edges of a finite phased array. With the use of diffraction theory and an extension of equivalent current method, the E_ϕ cross-polarized fields can be determined with good agreement and therefore can be used to predict in a range between $\theta = \pm 60°$. Therefore, phased array antennas with low cross-polarization requirements that have ground discontinuities can have diffraction fields presence in multiple sections of the array. Diffraction should be evaluated for individual elements and it is made possible with the proposed model.

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