Current Polarization Impact on Cross-Polarization Definitions for Practical Antenna Elements

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***Abstract*—With the growing interest in polarization diversity in communications and radar systems, the use of Ludwig's second and third definitions have become controversial among scientists and antenna engineers. Therefore, this paper is an attempt to clarify some of the ambiguity and confusion caused by these definitions. A detailed comparison of Ludwig's 2nd and 3rd definitions of cross-polarization, as applied to linearly polarized antennas, was performed. The results show that, in the diagonal plane, Ludwig's 2nd definition leads to a lower cross-polarization level than the 3rd definition for x- or y-polarized current sources. For a Huygens source, by Ludwig's 3rd definition, the radiation pattern has a lower cross-polarization level than that obtained by Ludwig's 2nd definition.**

I. Introduction

Recently, dual-polarized weather radars are gaining more popularity due to their ability to provide more accurate and timely weather observations (detection and measurement of precipitation). In general, a single polarized radar provides available information about the horizontal dimension of clouds and precipitation. However, dual-polarized radars significantly improve weather prediction by adding information about the vertical scattering off precipitation using the vertical polarization simultaneously or alternatively with the horizontal polarization (orthogonal polarization). In such weather radars, the antenna element used must also be dual-linearly polarized in two orthogonal planes, with a high degree of independence, since any error in measurement would lead to inaccurate weather data. Errors imay be created due to the interference between the cross-polar component of one of the ports and the co-polar component of the other port. In addition to mismatches in gain between H and V, cross polarization would affect the mutual coupling between the polarization ports. Therefore, reduction of cross-polarization is an important factor for high performance dual-polarized weather radar [1]-[4].

For phased array weather radar, the cross-polarization level is dramatically changed as its main beam is steered away from broadside direction, having the highest cross-polarization value in the diagonal planes. Designing a phased array antenna with acceptable cross-polarization isolation between the orthogonal antenna ports (H and V polarizations) over the whole scanning angle is the a primary challenge for any polarimetric weather radar.

For any antenna, the theoretical and measured radiation patterns are usually represented in terms of the vector components, specified by the unit vectors θ and ϕ in the spherical coordinate system. The radiated electric fields, as a function of direction, are specified by the angles θ and ϕ. These two components are aligned to cartesian unit vectors in the principal planes and at broadside. However, off the principal planes, the definition of co- and cross-polarization vector components and directions depends on how the polarization basis is defined. These components become coupled with each other when scanning off-broadside. The spatial angular relationship between radiated fields in the beam direction, off-broadside, is a matter of geometric projection of the field components.

It is well known that electromagnetic sources are usually described in terms of a cartesian coordinate system, while far-field radiation is described in terms of a spherical coordinate system with the same origin. However, using the same origin for both coordinate systems leads to some ambiguities and confusion in the use of the appropriate definition of co- and cross-polarization. Because of this, there is no universally accepted definition of cross-polarization.

Ludwig, in [5], discusses and presents three alternative definitions of the co- and cross-polarization as applied to linearly polarized antennas. These definitions are essentially the same in the principal planes, but they seriously disagree out of the principal planes and far away from broadside. Later, in a commentary for Ludwig's paper [6], the author mentions that the Ludwig-3 definition cannot be the standard definition of the cross-polarization, and it is not optimal for electric and magnetic dipoles, since they would have significant cross-polarization out of the principal planes. Also, a Huygens source would have no cross-polarization under the Ludwig-3 definition. In [7], a generalization of Ludwig's third definition is introduced by including a θ-dependence not involved in Ludwig's original definition. The controversy surrounding these definitions has caused some ambiguity and confusion regarding the use of the most meaningful description of cross-polarization, and not much work has been done to identify and clarify those definitions. In this paper, we are addressing this controversy by attempting to clarify the definitions using some linearly polarized antennas as examples. In addition, the cross- polarization levels of a wire and printed dipole, rectangular microstrip patch, and open-ended rectangular wave guide antenna have been used to illustrate Ludwig's definitions of cross polarization. Although cross polarization definitions are provided in the paper by Ludwig, there is no detailed mathematical derivation provided. Therefore, the antenna community is still confused when following the derivation in Ludwig's paper. Primarily for educational purposes, another objective of this paper is to provide a detailed formulation of

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Ludwig's definitions of the cross polarization using critical notes found in his manuscript [6].

According to [5], the cross polarization of an antenna, linearly polarized, can be defined in three ways:

Ludwig 1. Co- and cross-polarization unit vector directions coincide with that of a rectangular coordinate system

$$\hat{u}_{co} = \hat{y} = \sin\theta \sin\phi \; \hat{r} + \cos\theta \sin\phi \; \hat{\theta} + \cos\phi \; \hat{\phi} \tag{1}$$

$$\hat{u}_{cross} = \hat{x} = \sin\theta \cos\phi \; \hat{r} + \cos\theta \cos\phi \; \hat{\theta} - \sin\phi \; \hat{\phi} \tag{2}$$

Ludwig 2. A unit vector, tangential to a spherical surface, is considered as co-polarization and the direction of another unit vector, tangent to the spherical surface, corresponds to the cross- polarization. For an electric dipole polarized in the y direction, Ludwig 2-I definition is presented by

$$\hat{u}_{co} = \frac{\sin\phi \cos\theta \; \hat{\theta} + \cos\phi \; \hat{\phi}}{\sqrt{1 - \sin^2\phi \; \sin^2\theta}} \tag{3}$$

$$\hat{u}_{cross} = \frac{\cos\phi \; \hat{\theta} - \sin\phi \cos\theta \; \hat{\phi}}{\sqrt{1 - \sin^2\phi \; \sin^2\theta}} \tag{4}$$

And for an x-polarized antenna, Ludwig 2-II definition is presented by

$$\hat{u}_{co} = \frac{\cos\phi \cos\theta \; \hat{\theta} - \sin\phi \; \hat{\phi}}{\sqrt{1 - \cos^2\phi \; \sin^2\theta}} \tag{5}$$

$$\hat{u}_{cross} = \frac{\sin\phi \; \hat{\theta} + \cos\phi \cos\theta \; \hat{\phi}}{\sqrt{1 - \cos^2\phi \; \sin^2\theta}} \tag{6}$$

Ludwig 3. The co- and cross-polarization are defined based on measured radiation patterns of the antenna while tilting and rotating the antenna under test (AUT), rather than moving the probe antenna along a sphere, as in spherical near-field ranges. The unit vectors can be expressed in terms of the spherical coordinate unit vectors by

$$\hat{u}_{co} = \sin\phi \; \hat{\theta} + \cos\phi \; \hat{\phi} \tag{7}$$

$$\hat{u}_{cross} = \cos\phi \; \hat{\theta} - \sin\phi \; \hat{\phi} \tag{8}$$

These equations were derived based on the behavior of an ideal Huygens source composed of orthogonal electric and magnetic currents placed along the y and x axis, respectively.

II. Source current polarization

It is necessary to relate the polarization of the source current to the polarization of the far field radiated by these sources. The current distribution method is the most common technique used for calculating the co- and cross-polarization performance of an antenna with help of the *field equivalence principle* [8]. The derived equations using this method have the advantage of being expressed in terms of the current source polarization of an antenna in the x-y plane. By specifying the equivalent current densities, J_s and M_s of the antenna electric fields, E_θ and E_ϕ, the components of the far field can be determined.

A spherical coordinate system shown in Fig. 1 is used with the AUT in the x-y plane and the z-axis normal to the antenna. The polar angle θ and azimuth angle ϕ are measured as usual from the z-axis and x-axis, respectively. Applying the second and third definitions using the equations derived above, Table I summarizes the contributions of the electric and magnetic currents, oriented along the x and y axis, to the cross polarization in the far field patterns. It is apparent that the dominant cause of cross polarization is the x source current by Ludwig 2-I, and the y source current by Ludwig 2-II. If the third definition is used, the radiation patterns of the electric and magnetic currents, oriented in x and y directions, would have no cross-polarization in the main planes and significant cross-polarization out of the E- and H- planes and far away from broadside.

TABLE I
Source Current Contributions to Far-field Radiation Patterns Cross-Polarization

Definition	Direction	I^e	I^m
Ludwig 2-I	i_x	$\frac{\sin\phi\cos\phi\sin^2\theta}{\sqrt{F}}$	$\frac{\cos\phi\sin\phi\sin^2\theta}{\sqrt{F}}$
	i_y	0	0
Ludwig 2-II	i_x	0	0
	i_y	$\frac{\cos\phi\sin\phi\sin^2\theta}{\sqrt{F}}$	$-\frac{\cos\phi\sin\phi\sin^2\theta}{\sqrt{F}}$
Ludwig 3	i_x	$\frac{\cos\phi\sin\phi(1-\cos\theta)}{\sqrt{1-\cos^2\phi\sin^2\theta}}$	$\frac{\cos\phi\sin\phi(1-\cos\theta)}{\sqrt{1-\cos^2\phi\sin^2\theta}}$
	i_y	$\frac{\cos\phi\sin\phi(1-\cos\theta)}{\sqrt{1-\sin^2\phi\sin^2\theta}}$	$-\frac{\cos\phi\sin\phi(1-\cos\theta)}{\sqrt{1-\sin^2\phi\sin^2\theta}}$

$F = (1 - \sin^2\phi \sin^2\theta)(1 - \cos^2\phi \sin^2\theta)$

Similarly, equivalent current contributions of an aperture antenna to radiation pattern cross-polarization are summarized in Table II. It has been shown that an aperture antenna with a combination of orthogonal electric and magnetic currents (Huygens source) has a pattern with no cross-polarization by definition 3. For a Huygens source, the aperture electric and magnetic fields are related by the uniform plane wave relationship as illustrated in Fig. 2. This condition is not easily satisfied for aperture antennas. Therefore the Huygens source can be considered an approximation.

III. Simulation results

Four types of antenna were used to illustrate Ludwig's definitions. The commercial software HFSS [6] was used to design the antennas which consisted of wire and printed dipoles, a rectangular microstrip patch, and an open-ended rectangular waveguide. HFSS-simulated radiation patterns have been made at an operating frequency of 3 GHz for all antennas. Since the patterns are the same in the principal planes, Ludwig's 2nd and 3rd definitions are only compared in a 45-degree skewed plane.

A half-wavelength wire dipole antenna oriented along the y and x-axis, as shown in Figs. 2 and 3, is considered. The

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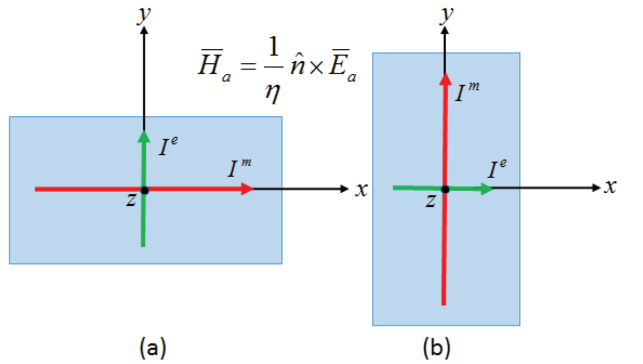

Fig. 1. Aperture antenna with a Huygens source oriented along a) y direction b) x direction

TABLE II
APERTURE CURRENT CONTRIBUTIONS TO FAR FIELD RADIATION PATTERNS CROSS-POLARIZATION

Definition	J_y & M_x	J_x & M_y
Ludwig 2-I	$\frac{\sin\phi\cos\phi(1-\cos\theta)}{\sqrt{1-\sin^2\phi\sin^2\theta}}$	$\frac{\cos^2\phi-\cos\theta\sin^2\phi}{\sqrt{1-\sin^2\phi\sin^2\theta}}$
Ludwig 2-II	$-\frac{\sin\phi\cos\phi(1-\cos\theta)}{\sqrt{1-\cos^2\phi\sin^2\theta}}$	$\frac{\cos^2\phi\cos\theta-\sin^2\phi}{\sqrt{1-\cos^2\phi\sin^2\theta}}$
Ludwig 3	0	0

co- and cross-polar radiation patterns in the D plane are calculated. For a y-polarized dipole, shown in Fig. 2, Ludwig 2-I yields zero cross-polarization. On the other hand, Ludwig 2-II gives a radiation pattern with zero cross-polarization for an x-polarized dipole shown in Fig. 3. These results are consistent with the results shown in Table I for electric current sources oriented in the y and x axis, respectively.

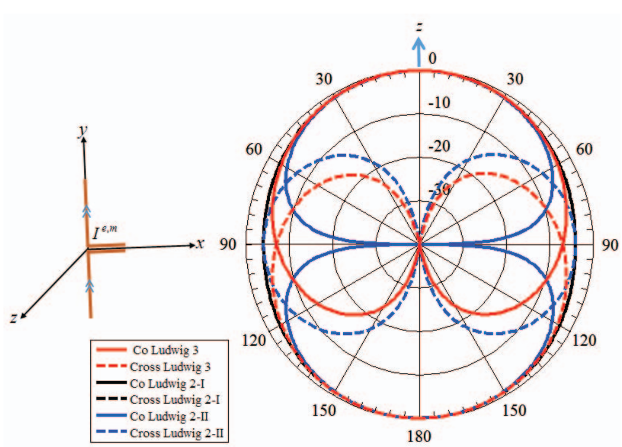

Fig. 2. $\lambda/2$ dipole antenna polarized in a) y direction b) x direction

Figures 4 and 5 illustrate the co- and cross-polarized patterns in the D-plane of a rectangular microstrip patch antenna polarized in the y and x directions, respectively. It is demonstrated that the patch antenna polarized in y direction,

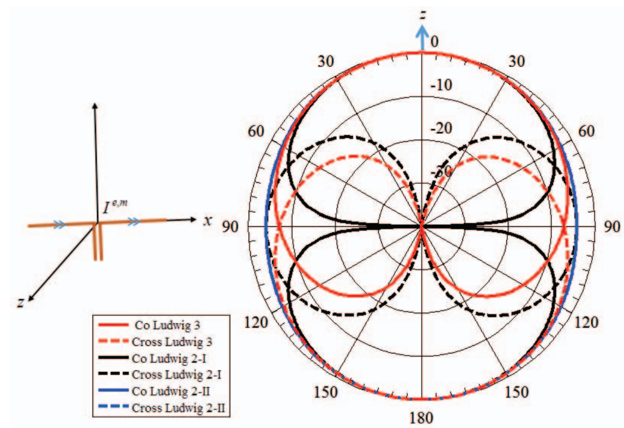

Fig. 3. Co- and cross-polarized radiation patterns in the D-plane of a rectangular microstrip patch antenna polarized in a) y direction b) x direction

as shown in Fig. 4, has a lower cross polarization level, by Ludwig 2-II, compared to the other two definitions. On the other hand, the x-polarized patch antenna, as shown in Fig. 5, has a better cross-polarization level by Ludwig 2-I definition. This is consistent with the earlier analysis shown in Table I.

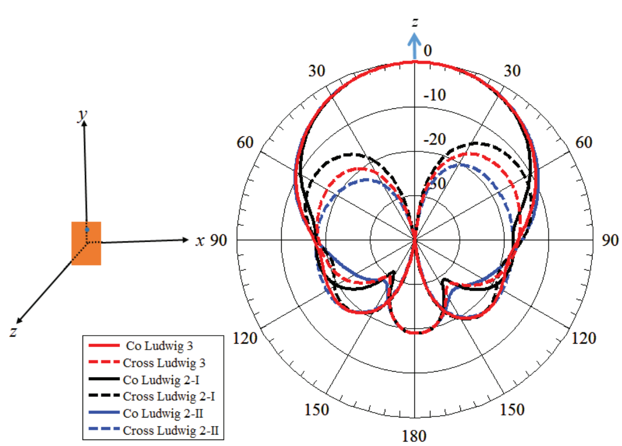

Fig. 4. Co- and cross-polarized radiation patterns in the D-plane of a rectangular microstrip patch antenna polarized in a) y direction b) x direction

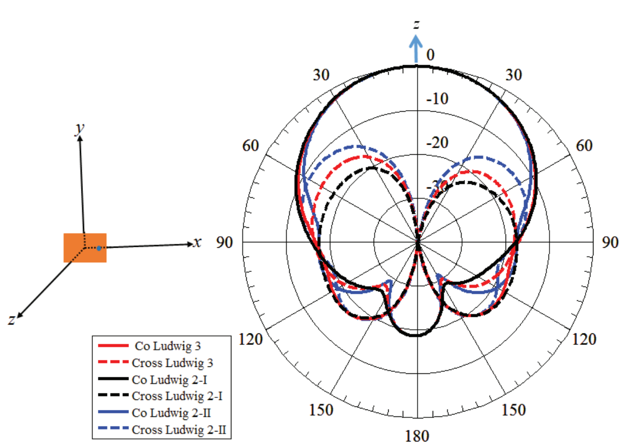

Fig. 5. Rectangular patch antenna polarized in a) y direction b) x direction

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The open-ended rectangular waveguide (OEWG) is excited in the dominant TE10 mode of operation. This antenna provides a close approximation of a Huygens source. If the aperture electric field of the TE10 mode is polarized in the y direction, as illustrated in Fig. 6, the aperture has an equivalent magnetic current in the y direction and an equivalent electric current in the x direction. The same currents, with different directions over the aperture, are obtained if the antenna aperture is polarized in the x direction, as illustrated in Fig. 7. In both cases, the Ludwig-3 definition gives lower cross-polarization levels, as shown in Fig. 8. It is also shown that one of the definitions gives a close result to that obtained by the Ludwig-3 definition, because the antenna is not a perfect Huygens source. The aperture of the OEWG antenna has orthogonal equivalent electric and magnetic currents with unequal strengths.

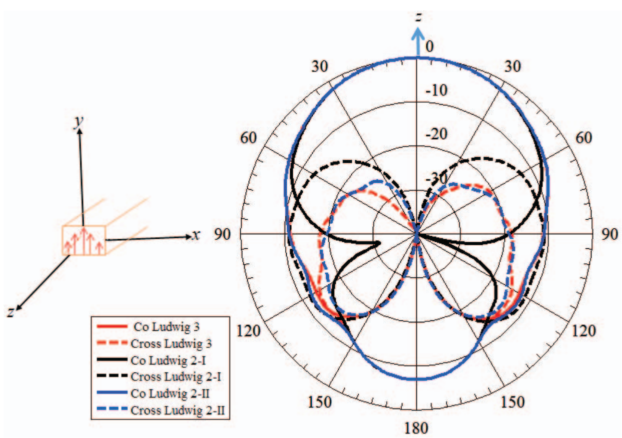

Fig. 6. Co- and cross-polarized radiation patterns in the D-plane of a rectangular microstrip patch antenna polarized in a) y direction b) x direction

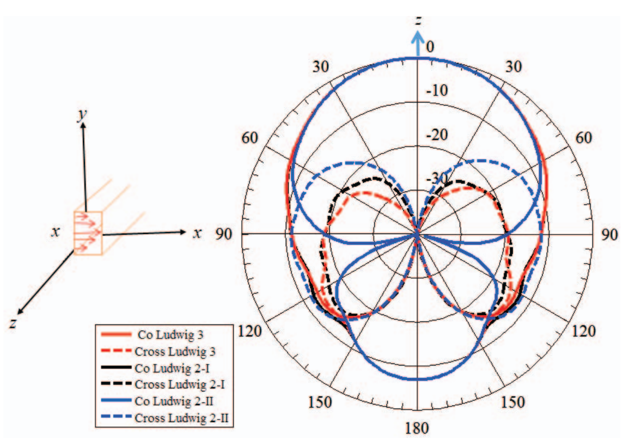

Fig. 7. Co- and cross-polarized radiation patterns in the D-plane of a rectangular microstrip patch antenna polarized in a) y direction b) x direction

The co- and cross-polar components of the printed dipole antenna for two different orientations in the D-plane are calculated using Ludwig's definitions. It is apparent that Ludwig 2-I definition gives a low cross-polarization level of the y-polarized dipole antenna in Fig. 8. However, the x-polarized dipole antenna, as shown in Fig. 9, has a lower cross-polarization level by the Ludwig 2-II definition. These results are consistent with the earlier analysis in Table I.

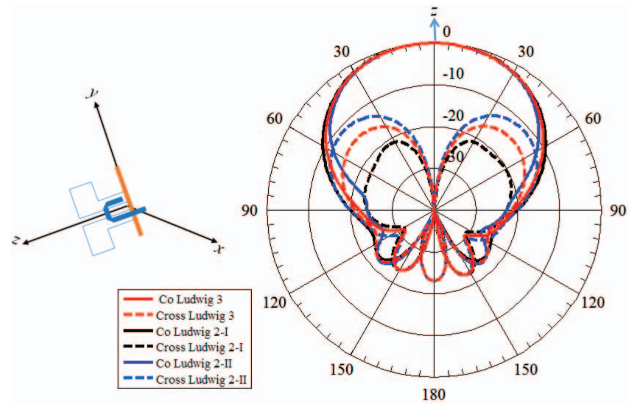

Fig. 8. Co- and cross-polarized radiation patterns in the D-plane of an ended-open rectangular waveguide antenna polarized in a) y direction b) x direction

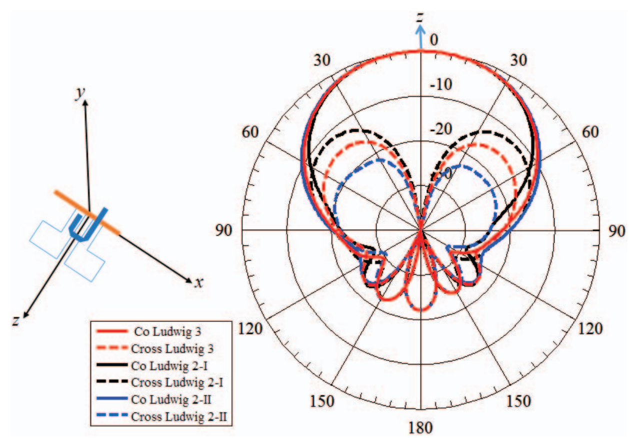

Fig. 9. Co- and cross-polarized radiation patterns in the D-plane of an printed dipole antenna polarized in a) y direction b) x direction

IV. Conclusions

The effects of aperture polarization control on the far field cross-polarization patterns of the antenna have been described. In the diagonal plane, Ludwig's 2nd definition leads to a lower cross-polarization levels than the 3rd definition for x- or y-polarized current sources. For a Huygens source, by Ludwig's 3rd definition, the radiation pattern has a lower cross-polarization level than that obtained by the Ludwig's 2nd definition.

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