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Two-Dimensional Beam Pattern Synthesis for Phased Arrays With Arbitrary Element Geometry via Magnitude Least Squares Optimization

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ABSTRACT In this paper, an optimization procedure for synthesizing shaped beams with an arbitrary geometry phased array system with real-time capability is presented. The algorithm involves a combination of an arbitrary extension of the Woodward-Lawson synthesis procedure with the magnitude least-squares optimization method. This simple combination significantly reduces the optimization time, enabling real-time beam shaping in response to changing requirements on beam shape or evolving geometry in distributed arrays. The technique places no restrictions on element positioning in all three spatial dimensions and is demonstrated to accurately reproduce the desired beam shape in both angular dimensions. The algorithm is demonstrated for multiple beam cases with a large, randomly generated array in computational simulation. The applicability of the algorithm to practical phased array hardware is also demonstrated using full-wave electromagnetic simulations and measurements of a realized arbitrary phased array system using a near-field scanner in an anechoic chamber.

INDEX TERMS Phased arrays, array pattern synthesis, beam-forming, conformal arrays, distributed arrays, magnitude least-squares optimization.

I. INTRODUCTION

A significant advantage of phased array systems is the electronic reconfigurability of the beam pattern, enabling quick changes to the direction of transmission in real-time. While many applications require the generation of pencil beams steered in a single desired direction, other applications in radar and communications may require beam shapes of arbitrary configuration. Procedures for producing these beam shapes are referred to as synthesis techniques, and they are numerous. Several traditional synthesis methods exist for beam pattern synthesis when the array is planar and periodic. These include techniques such as the Schelkunoff Polynomial method [1], Fourier synthesis [2], [3], and Woodward-Lawson synthesis [4], [5]. More recent methods for planar array synthesis have also been proposed leveraging theoretical developments [6], [7] and optimization procedures [8]–[11].

However, these planar array methods are no longer applicable in situations where the array element geometry is aperiodic and non-planar. Such a geometry may be unavoidable in some applications, such as with conformal or distributed arrays. Analytical methods for synthesizing a desired beam shape in this case are difficult or impossible to develop; therefore, the majority of solutions proposed for this problem involve some form of optimization. For example, algorithms have been proposed for arbitrary array beam shaping based on genetic algorithms [12], convex semidefinite relaxation [13], iterative least-squares [14], and linear programming [15], among others. These optimization procedures often lead to large computational burdens, and as such they do not lend themselves to real-time beam shaping applications. This would be particularly deleterious in a distributed RF application when the elements forming the array are mobile, leading to non-constant geometries and requiring frequent re-computation of the element weights.

This paper proposes a combination of the magnitude least-squares (MLS) optimization algorithm proposed in [16] with an extension of the Woodward-Lawson synthesis technique developed by the authors to accommodate the arbitrary array geometry. The algorithm works by generating a first-order guess of the required array excitations using the Woodward-Lawson extension and then using these excitations as the initial weights in the MLS optimization. The resulting optimization enables the formation of the desired beam pattern with high accuracy and requires substantially fewer iterations than the case with uniform unit initial weights, enabling real-time beam shaping even in cases with a large number of elements. The real-time potential for this algorithm would make it useful for real-time beam shaping with non-planar arrays such as a conformal array system, and would also be useful for beam shaping in situations when the geometry of the array is not constant, such as in an array composed of a large set of mobile sensor elements [17].

The primary contribution of the proposed beam shaping technique lies in its simplicity and real-time capability. Speed is not a priority for other techniques proposed in the literature for beam pattern formation with arbitrary arrays, and as such information on computation time is often omitted, casting doubt on whether these techniques may be utilized in situations where the beam pattern or array geometry must change in real-time. In cases where computation time is explicitly included (such as [15]), the technique proposed here is shown to be faster. Moreover, existing techniques provided in the literature are often only demonstrated for one angular dimension and seldom, if ever, are the techniques validated by measured results of a physically constructed arbitrary array. To demonstrate both the complete flexibility of the proposed technique in this article and its application in real hardware, shaped beam patterns are formed in both angular dimensions using randomly generated arbitrary arrays, and the technique is proven in practice using a physical arbitrary array measured using a near-field scanner.

This paper is structured as follows. In Section II, the mathematical components of the algorithm are presented. In Section III, simulated results are provided to demonstrate the algorithm's effectiveness in several situations. Full-wave electromagnetic simulations and near-field scanner measurements of a realized arbitrary phased array are presented in Section IV to validate the algorithm's applicability to real hardware. Conclusions on the algorithm performance are drawn in Section V.

II. ALGORITHM DESCRIPTION

Most traditional synthesis techniques rely on the geometry of the array being planar and periodic, and as such in their derived forms they are not applicable to the problem of beam pattern synthesis with arbitrary array geometry. However, an approximation of the Woodward-Lawson synthesis may be formulated to generate a good first-order approximation of the required excitations. These approximate values may be used as the starting point for the primary optimization procedure forming the core of the beam shaping algorithm.

A. WOODWARD-LAWSON SYNTHESIS

Woodward-Lawson synthesis works as follows. Recall that the sine-space variables u and v are related to the elevation angle θ and the azimuth angle ϕ by

$$\begin{aligned} u &= \sin\theta\cos\phi \\ v &= \sin\theta\sin\phi. \end{aligned} \tag{1}$$

Suppose that it is desired to synthesize an array factor pattern $F(u, v)$ where the array factor is separable in u and v, i.e. $F(u, v) = F(u)F(v)$. Suppose that the array is a rectangular $N_x \times N_y$ planar array with x and y element spacing d_x and d_y, respectively. The full array factor may be sythesized by determining the x-dimension element weights $\mathbf{a}_x$ to form $F(u)$ and the y-dimension element weights $\mathbf{a}_y$ to form $F(v)$ and then determining the weights for each individual element $\mathbf{A}_{xy}$ by

$$\mathbf{A}_{xy} = \mathbf{a}_x\mathbf{a}_y^T. \tag{2}$$

The element weights in each dimension may therefore be solved for independently in an identical manner. To synthesize $F(u)$, the desired pattern is divided into composing functions. Each composing function is a beam steered to a point $u = u_i$ by computing the element weights

$$a_x^i(n_x) = e^{-jk_0 d_x u_i n_x} \tag{3}$$

where

$$n_x = \begin{cases} \pm 1/2, \pm 3/2, \pm 5/2 \ldots & N_x \text{ even} \\ 0, \pm 1, \pm 2, \pm 3 \ldots & N_x \text{ odd} \end{cases} \tag{4}$$

and $k_0 = 2\pi/\lambda$ and λ is the operating wavelength. The values of u_i are computed by

$$u_i = \frac{\lambda}{N_x d_x} i \tag{5}$$

where the values of the index i take on the same values as n_x in (4). The composing functions steered to u_i lead to each composing function having a beam maximum in the nulls of each other composing function, leading to orthogonality. Because of this orthogonality, each composing function and its associated element weights can be scaled by the value of $F(u_i)$ and then summed together to give the total weights required to synthesize the function, giving

$$a_x(n) = \sum_i F(u_i)a_x^i(n). \tag{6}$$

The process is similar to sinc interpolation in signal processing, where sinc functions at each discrete sample are weighted and added together to produce the desired continuous-time signal from discrete values. Similarly, the vertical element weights $a_y(n)$ may be formed by placing composing functions at v_k, where the values of v_k are computed similarly to the values of u_i in (5).

Note that although the formulation of this synthesis procedure is simpler if $F(u, v)$ is separable in u and v, this need not be the case. In the case of a more complex function $F(u, v)$, a two-dimensional grid of composing functions must be formed, rather than in one dimension at a time, and the weights must be computed as a function of both the x and y coordinate of the elements.

B. EXTENSION OF WOODWARD-LAWSON SYNTHESIS FOR ARBITRARY GEOMETRY ARRAYS

As mentioned previously, the Woodward-Lawson synthesis method for planar periodic arrays is useful to understand as it provides the basis for a very simple (but also fairly poorly-performing) algorithm for generating a shaped beam with an arbitrary array. Recall that the general array factor for an arbitrary geometry array of N elements whose element positions are given by position vectors $\bar{r}_n$ is given by

$$F(\theta, \phi) = \sum_{n=1}^{N} a_n e^{jk_0 \hat{r} \cdot \bar{r}_n} \tag{7}$$

assuming that the element pattern for each element is identical and $\hat{r}$ is the position vector of points in the far-field given by

$$\begin{aligned} \hat{r} &= \hat{x} \sin\theta \cos\phi + \hat{y} \sin\theta \sin\phi + \hat{z} \cos\theta \\ &= \hat{x}u + \hat{y}v + \hat{z} \cos\theta. \end{aligned} \tag{8}$$

As in a periodic array, it is possible to compute the unit magnitude complex excitations a_n for all elements to steer the beam of an arbitrary array to point in an arbitrary direction given by $\hat{r}_0$. The direction $\hat{r}_0$ is given by

$$\begin{aligned} \hat{r}_0 &= \hat{x} \sin\theta_0 \cos\phi_0 + \hat{y} \sin\theta_0 \sin\phi_0 + \hat{z} \cos\theta_0 \\ &= \hat{x}u_0 + \hat{y}v_0 + \hat{z} \cos\theta_0. \end{aligned} \tag{9}$$

where θ_0 and ϕ_0 (and by extension u_0 and v_0) are the desired angles toward which to steer the beam. Given $\hat{r}_0$, the complex excitations a_n are given by

$$a_n = e^{-jk_0 \hat{r}_0 \cdot \bar{r}_n}. \tag{10}$$

Similarly to the steered beams in Woodward-Lawson synthesis, these steered beams evaluated for different values of u_0 and v_0 may be considered composing functions for the array pattern, and as such the a_n values used to generate such beams may be added together to produce new complex weights for forming shaped beams. However, these new arbitrary array composing functions differ from the planar periodic composing functions in significant ways. Because the elements are not spaced in any regular way, it is not immediately obvious which values of u and v should be selected to evaluate $F(\theta, \phi)$.

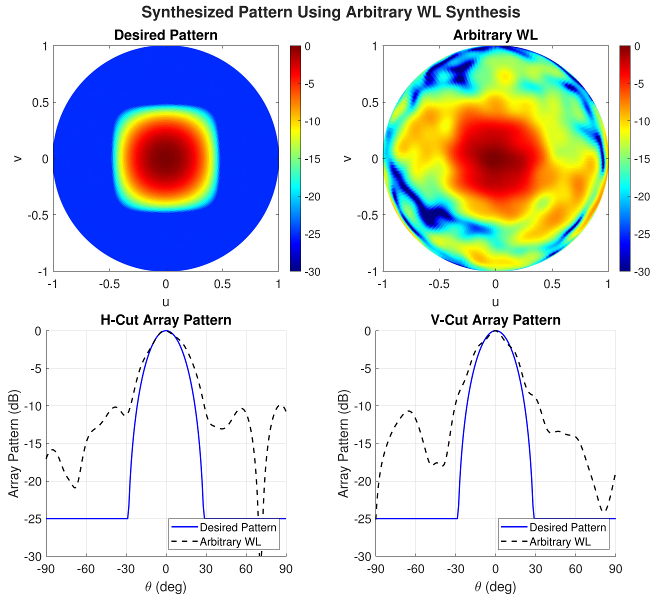

FIGURE 1. The desired pattern (top left), the pattern synthesized using the arbitrary Woodward-Lawson synthesis (top right), and the H- and V-cuts of the pattern (bottom left and right).

Because of this, it is deemed simplest to select linearly spaced values u_i and v_k where

$$0 \le i, k < \left\lceil \sqrt{N} \right\rceil \tag{11}$$

and where the complex excitations used to steer the beam in the (u_i, v_k) direction are denoted as a_n^{ik}. These composing element excitations are computed by

$$a_n = e^{-jk_0 \hat{r}_{ik} \cdot \bar{r}_n} \tag{12}$$

where

$$\hat{r}_{ik} = \hat{x}u_i + \hat{y}v_k + \hat{z} \cos\theta_{ik} \tag{13}$$

and θ_{ik} is computed by

$$\theta_{ik} = \sin^{-1}\left(\sqrt{u_i^2 + v_k^2}\right). \tag{14}$$

There is no guarantee that these composing functions will be regular sinc functions due to the irregularity of the array, so different parts of the array pattern may not be composed uniformly. Furthermore, there is no way to ensure orthogonality of the composing functions, so when the complex weights for each functions are added together, the composing functions will interfere with one another and degrade the array pattern. Given the composing excitations for each element, the total complex excitations for each element are then given by

$$a_n = \sum_i \sum_k F(u_i, v_k) a_n^{ik} \tag{15}$$

where the element weights are usually normalized such that the maximum magnitude is 1. To see how this method works, consider the beam patterns shown in Fig. 1. The desired pattern is a sinc function centered at the origin of the (u, v) space

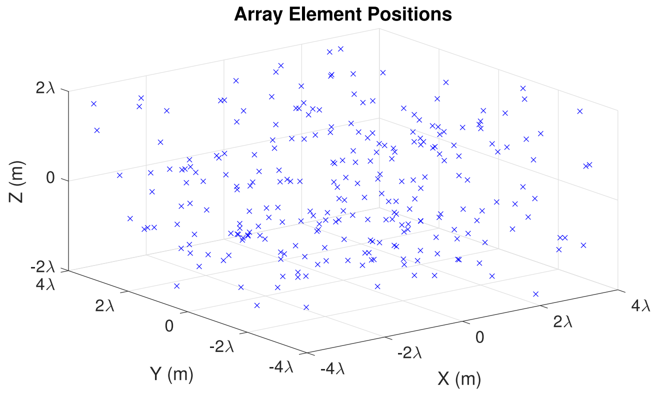

FIGURE 2. The element positions for the simulated array used to generate the beam pattern in Fig. 1.

with the first nulls at 30 degrees ($u = v = 0.5$) and the desired sidelobe level set to a constant -25 dB after the location of the first null. The arbitrary Woodward-Lawson synthesis is used to compute the element weights for 256 randomly generated uniformly within an $8\lambda \times 8\lambda \times 4\lambda$ space. The element positions for this array are shown in Fig. 2. Observe how the sidelobe levels of the generated pattern are quite high, and the general shape of the pattern matches somewhat to the desired pattern but with significant deformities. These results indicate that the arbitrary Woodward-Lawson synthesis does not work well on its own owing to the non-orthogonality of the composing functions that leads to significant synthesis errors. However, this technique works well at generating "first-guess" element weights for an optimization procedure, which is discussed next.

C. MAGNITUDE LEAST SQUARES OPTIMIZATION

Given the non-uniform and unpredictable nature of the arbitrary array, it is difficult and perhaps impossible to arrive at any closed-form solution for the optimal complex element weights to produce a desired array pattern. Therefore, the solution to this problem lies in some form of optimization algorithm, such as least-squares optimization. The goal of least-squares optimization is to find the values of a length-N vector β that minimize the sum

$$S = \sum_{m=1}^{M} (f(x_m, \beta) - y_m)^2 \tag{16}$$

where y_m is the m^{th} element of a vector $\mathbf{y}$ and is the "desired" function value at point m, and $f(x_m, \beta)$ is the realized function value at point m parameterized by the vector β. If $f(x_m, \beta)$ is a linear function of the elements of β, then (16) may be rewritten as

$$S = \sum_{m=1}^{M} (A_m \beta - y_m)^2 \tag{17}$$

where A_m are rows of an $M \times N$ matrix $\mathbf{A}$.

In (7), it is seen that the array factor of the arbitrary array is a linear sum of the coefficients a_n multiplied by the phase factor in the complex exponential. Because the phase factor will be constant when evaluated for a given angle (u_m, v_m) and excitation a_n, the phase factors can be expressed as the values of the matrix $\mathbf{A}$. In this case, each entry A_{mn} is given by

$$A_{mn} = e^{jk_0 \hat{r}\cdot\bar{r}_n}\Big|_{u=u_m, v=v_m} \tag{18}$$

where the pairs (u_m, v_m) are now indexed by a single variable m, but should still be linearly spaced in a grid to cover the full (u, v) space. Note that in some cases such as conformal arrays or distributed arrays composed of dissimilar antenna elements, the array pattern must be optimized in terms of the individual element patterns. In this case, the entries of A_{mn} may be given by

$$A_{mn} = f_n(u, v) e^{jk_0 \hat{r}\cdot\bar{r}_n}\Big|_{u=u_m, v=v_m} \tag{19}$$

where $f_n(u, v)$ is the element pattern of antenna element n. In this paper, the cases investigated assume that the element patterns are uniform, so A_{mn} are computed using the simple case in (18).

The vector β can be replaced by the vector of excitation weights $\mathbf{a}$. The value of y_m is then simply the desired magnitude of the array factor at (u_m, v_m), $F(u_m, v_m)$. Therefore, the least-squares formula may be rewritten as

$$S = \sum_{m=1}^{M} (A_m \mathbf{a} - F(u_m, v_m))^2. \tag{20}$$

In general, using the least-squares estimator, the optimal weights in $\mathbf{a}$ for minimizing S will be given by

$$\mathbf{a} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{y} \tag{21}$$

with

$$\mathbf{y} = \begin{bmatrix} F(u_1, v_1) \\ \vdots \\ F(u_M, v_M) \end{bmatrix}. \tag{22}$$

However, there is a dilemma: the least-squares minimization of the sum S assumes that the difference between $A_m \mathbf{a}$ and y_m is real. Although the value of y_m will be real assuming that the pattern synthesis is concerned with magnitude patterns, in general the value of $A_m \bar{a}_n$ is complex, and the magnitude difference between a complex and a real number is not well-defined. Theoretically, we may take the magnitude of this complex number, and the minimizing sum will then be given by

$$S = \sum_{m=1}^{M} (|A_m \mathbf{a}| - F(u_m, v_m))^2 . \tag{23}$$

However, this is problematic as the absolute value operator causes the function to no longer be linear. A clever reformulation of the problem is given in [16], [18] called the variable exchange method (VEM). In the VEM, the minimizing sum is rewritten as

$$S = \sum_{m=1}^{M} |A_m \mathbf{a} - F(u_m, v_m) z_m|^2 . \tag{24}$$

where z_m is the m^{th} element of a vector $\mathbf{z}$ and z_m is a unit magnitude complex number whose phase is set to be identical to the phase of $A_m\mathbf{a}$. That is,

$$z_m = e^{j\angle(A_m \mathbf{a})}. \tag{25}$$

Thus, the difference between $A_m\mathbf{a}$ and $F(u_m, v_m)z_m$ reduces to the difference between two phasors with the same phase, allowing the magnitude difference to be extracted by taking the absolute value of the whole expression. The squared value of the difference is then the magnitude squared of the resulting complex number. To give the optimal weights $\mathbf{a}$, the ordinary least-squares estimator need only be modified to

$$\mathbf{a} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{b} \tag{26}$$

where the superscript ^{H} denotes the complex transpose of a matrix and the elements of $\mathbf{b}$ are given by $b_m = F(u_m, v_m)z_m$.

To iteratively improve the array factor resulting from the selection of $\mathbf{a}$, the following steps should be followed:

1) Choose an initial set of weights $\mathbf{a}$ either by setting the initial weights as unit magnitude with zero phase or by using the arbitrary geometry extension of the Woodward-Lawson technique. For larger arbitrary arrays, the performance may be substantially increased and the number of required iterations decreased by using the extended Woodward-Lawson technique.
2) For all values of m, set z_m as the unit magnitude complex number with the same phase as $A_m\mathbf{a}$.
3) Compute new weights $\mathbf{a}$ using (26).
4) Find new z_m and recompute $\mathbf{a}$ until a certain number of iterations is reached or the error becomes sufficiently small.

This full optimization algorithm is referred to as the magnitude least-squares (MLS) algorithm [16].

The final consideration is the selection of points (u_m, v_m). The formulation of the problem does not easily allow for a three-dimensional optimization, and as such, A must be constructed using m as the independent variable rather than u and v. Therefore, a discrete set of pairs (u_m, v_m) must be selected for use in computing $\mathbf{A}$, with the values of (u_m, v_m) distributed over the entire (u, v) plane within the unit circle, or at least over the region to be optimized if not the full (u, v) unit circle. If too many pairs of (u_m, v_m) are used, the computation time for a reasonable number of iterations will be significant. However, if not enough pairs are used, then the optimized may have "gaps" in between the selected (u_m, v_m) points where the beam is not shaped as desired. Thus, there is a trade-off that must be made between improved accuracy and computation speed.

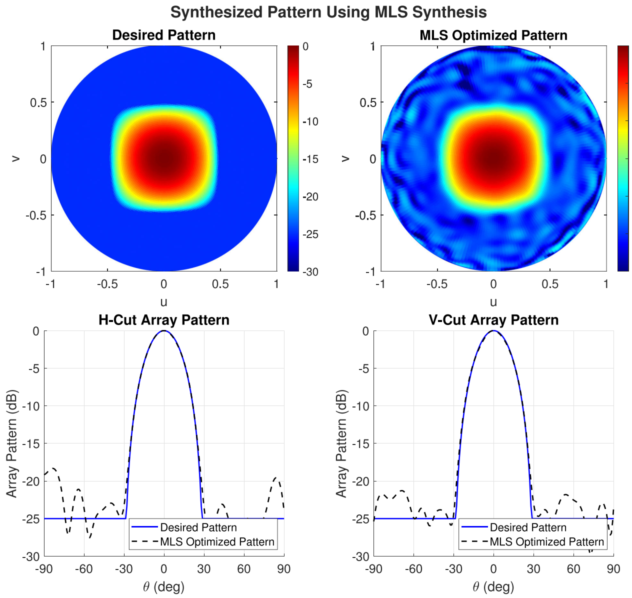

FIGURE 3. The desired pattern (top left), the pattern synthesized using the MLS optimization synthesis (top right), and the H- and V-cuts of the pattern (bottom left and right).

III. ANALYTICAL SIMULATION RESULTS

The proposed optimization algorithm is utilized in a simulation to validate its feasibility and demonstrate its effectiveness. The first pattern is formed by optimizing the zero-sidelobe sinc pattern first shown in Fig. 1 utilizing the same 256 element array given in Fig. 2. For the optimization procedure, 1000 iterations are performed and the initial weights are selected to be those produced using the arbitrary Woodward-Lawson synthesis. The optimization is performed using 8361 points, i.e. $M = 8361$. The simulated pattern results are shown in Fig. 3.

It is clear that the MLS optimized results in Fig. 3 are of much higher quality than those obtained using only the arbitrary Woodward-Lawson technique in Fig. 1. In addition to the synthesized beam shape much more closely matching the desired pattern, the sidelobes are significantly lower, particularly in the region close to the main beam, being nearly 10 dB lower at most points than the arbitrary Woodward-Lawson pattern.

A. IMPACT OF INITIAL WEIGHT SELECTION

The effectiveness of the proposed optimization procedure stands out much more when compared to the optimization procedure without the arbitrary Woodward-Lawson synthesis used to generate the initial guess at the element weights. When using other initial weights, such as setting all the initial weights to 1, the resulting patterns are significantly worse and the optimization procedure requires substantially more iterations to converge.

For instance, consider the beam patterns shown in Fig. 4. The desired beam pattern in this case is identical to that in

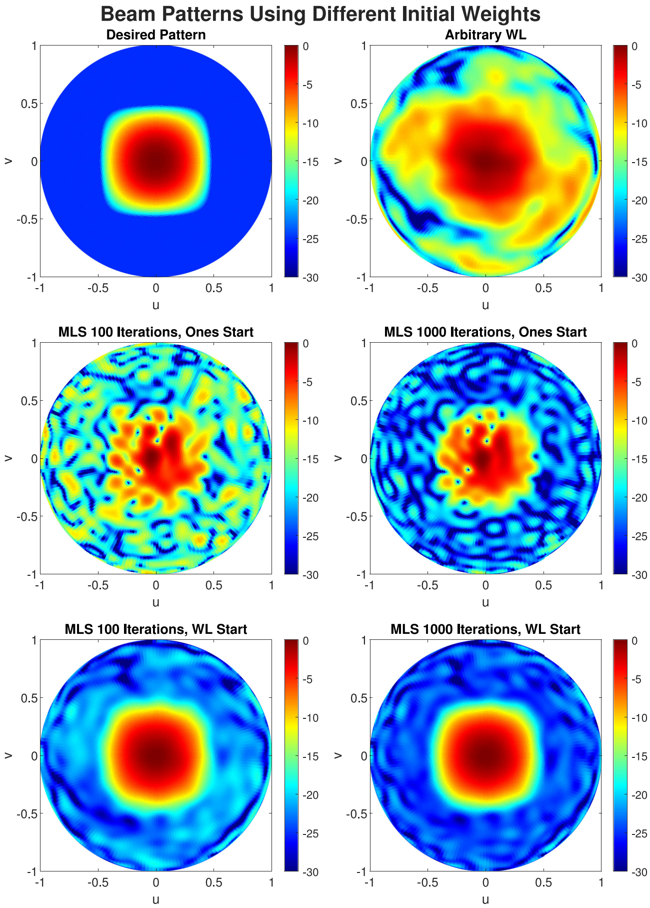

FIGURE 4. **The desired sinc pattern (top left), the pattern synthesized using the arbitrary Woodward-Lawson synthesis (top right), and the patterns synthesized using variations of the MLS optimization synthesis technique (center and bottom).**

Figs. 1 and 3, as are the simulated element positions. Four different patterns are formed using the MLS optimization procedure. Two are formed using uniform unit weights as the initial weights for the optimization procedure, and two are formed using the weights output by the arbitrary Woodward-Lawson technique as the initial weights. For each of these pairs, the patterns are formed using 100 iterations and 1000 iterations of the optimization. Qualitatively, it is clear that the additional iterations yields more accurate beam patterns compared to the ideal case. It is also clear that the patterns formed using the arbitrary Woodward-Lawson technique to give the initial optimization weights are of higher quality than those which use unit weights for the initial weights.

Quantitatively, the patterns formed with the arbitrary Woodward-Lawson initial weights also perform better than the unit initial weights. Table 1 lists the average error in dB of each pattern shown in Fig. 4 along with the amount of time required to produce the weights. From the numerical results, it is seen that using the arbitrary Woodward-Lawson technique improves the average error enough that a better pattern may be obtained using only a fraction of the iterations. For instance, the optimized pattern using the arbitrary Woodward-Lawson

TABLE 1. **Error and Convergence Time for the MLS Algorithm Under Different Conditions**

	Average Error (dB)	Time (s)
Arbitrary WL	7.1	0.1
MLS 100 Iters, Ones	6.1	4.1
MLS 1000 Iters, Ones	3.3	41.8
MLS 100 Iters, WL	2.3	4.3
MLS 1000 Iters, WL	1.3	41.6

initial weights after 100 iterations has less error than the pattern formed using 1000 iterations with unit initial weights (average error of 2.3 dB compared to 3.3 dB), leading to a higher quality of beam pattern synthesis with only $1/10^{\text{th}}$ of the computation time (computation time of 4.3 seconds compared to 41.8 seconds). These results show that even for a large array of 256 elements, a beam pattern may be recomputed on the order of seconds in response to changing pattern requirements. For smaller arrays, the computation time may easily be reduced to less than one second. If a particular desired average error level is specified, the optimization can be shown to reach the error bound much quicker when the arbitrary Woodward-Lawson weights are used; for instance, in this same pattern, to reach an average error of 5 dB, the optimization requires 181 iterations if the weights are set uniformly to 1 initially while only 14 iterations are required if the initial weights are found using the arbitrary Woodward-Lawson synthesis. In this sense, the proposed algorithm runs extremely quickly and if the arbitrary array has a reasonable number of elements, this algorithm may be employed to compute array element weights in real time in response to changes in the beam pattern requirement.

It is also worth noting that the improvement in speed and quality using the arbitrary Woodward-Lawson initial weights is only significant when the number of arbitrary elements is large. In smaller arrays, the results produced in either case will be similar, and for some basic patterns, such as a basic sinc function, using unitary initial weights will produce slightly better results.

B. SYNTHESIS OF COMPLEX BEAM PATTERNS

One advantage of the proposed algorithm is that it can synthesize element weights required to form arbitrary beam shapes in azimuth and elevation and does not require the patterns to be separable in u and v. So long as a sufficient number of elements are available, any number of complex beam shapes may be synthesized. For example, consider the complex beam shape in (u, v) shown in Fig. 5. These patterns are simulated using the array whose element positions are given in Fig. 2. In this case, the complex beam pattern causes the arbitrary Woodward-Lawson pattern synthesis to break down significantly in quality. However, it still provides a sufficient starting point for the optimized element weights, enabling a quality beam shape to be formed during the optimization procedure in a similarly short amount of time to the simpler sinc pattern case.

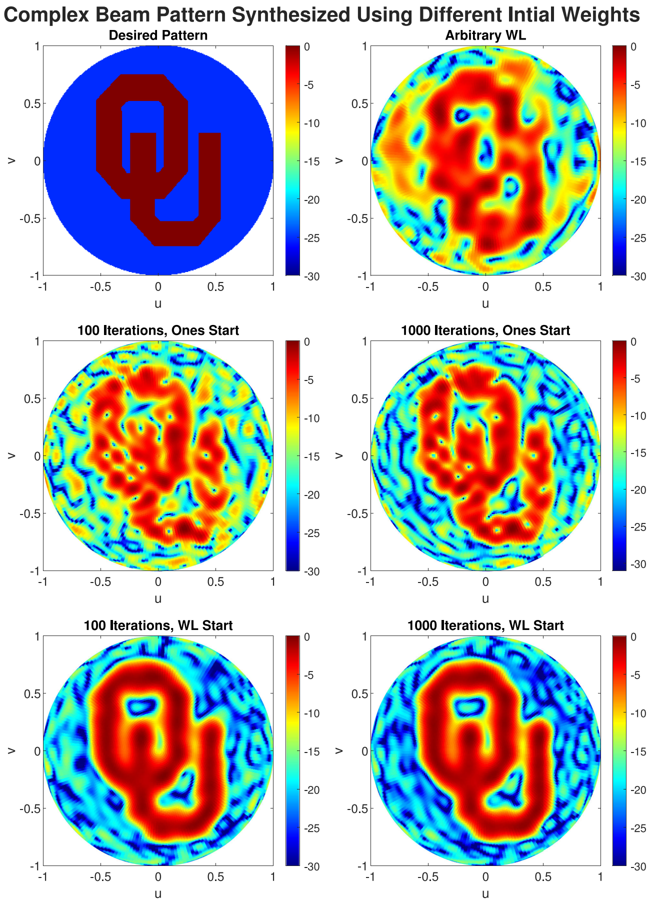

FIGURE 5. The desired complex pattern (top left), the pattern synthesized using the arbitrary Woodward-Lawson synthesis (top right), and the patterns synthesized using variations of the MLS optimization synthesis technique (center and bottom).

IV. EM SIMULATION AND MEASURED RESULTS

The proposed algorithm is validated in additional full-wave electromagnetic (EM) simulations using Ansys High-Frequency Structure Simulator (HFSS) and also with measured results performed using a custom-built arbitrary array developed at the Advanced Radar Research Center (ARRC) at the University of Oklahoma. A 16-channel amplitude and phase control (APC) board was used to define proper amplitude and phase of each element.

A. ARRAY GEOMETRY AND BEAM PATTERN

A custom-built phased array controller with 16 channels was available for the near-field measurements. As a result, a maximum of 16 elements may be used to form the arbitrary array, putting some restrictions on the complexity of the possible beam shapes. Therefore, a simple sinc function pattern was selected, with the beam pattern specified by

$$F(u) = |\text{sinc}\,(2(u + 0.2))|$$
$$F(v) = |\text{sinc}\,(2(v + 0.2))|$$
$$F(u, v) = F(u)F(v). \tag{27}$$

TABLE 2. The Element Configurations for the 16-Element Array

	x_n (cm)	y_n (cm)	z_n (cm)	$\lvert a_n \rvert$ (dB)	$\angle a_n$
1	15.0	30.0	8.7	-3.4	145.5°
2	15.0	22.5	9.6	-2.4	82.1°
3	10.0	30.0	9.3	-3.8	102.5°
4	10.0	25.0	8.3	-1.1	82.0°
5	5.0	22.5	8.2	-0.2	32.5°
6	5.0	27.5	8.5	0.0	83.0°
7	0.0	30.0	6.9	-2.3	86.2°
8	0.0	25.0	9.1	-1.0	17.4°
9	17.5	17.5	11.9	-8.2	-38.2°
10	15.0	12.5	9.3	-8.3	14.0°
11	10.0	12.5	10.4	-7.8	-85.1°
12	10.0	17.5	10.9	-1.3	-19.8°
13	5.0	10.0	12.0	-20.1	-169.7°
14	5.0	15.0	10.0	-1.8	-62.0°
15	0.0	12.5	10.0	-4.2	-102.7°
16	0.0	20.0	8.4	-1.1	-21.4°

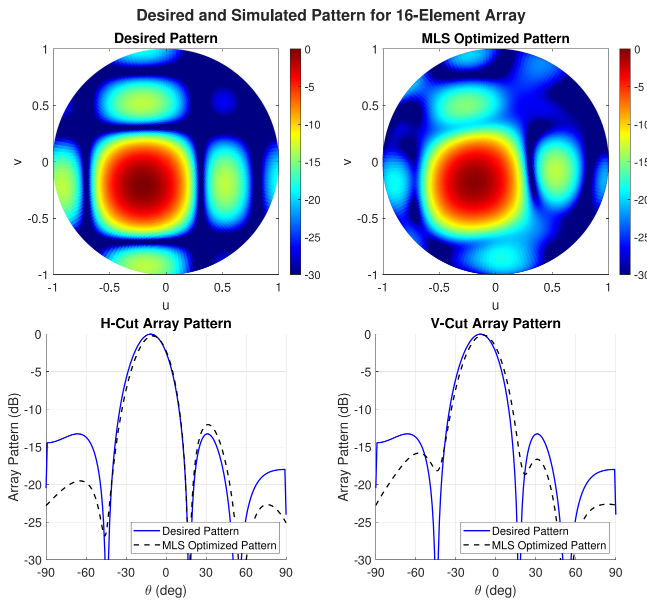

FIGURE 6. The ideal and synthesized array patterns for the 16-element array, in the full (u, v) space and in H and V cuts.

The elements are defined semi-randomly along a 2.5 cm grid in the x- and y-dimensions in accordance with the design of the element mounting hardware, with the z-position of each element being selected randomly. The element positions, along with the element excitations determined using the MLS optimization procedure, are shown in Table 2. The ideal pattern from (27) and the pattern synthesized by this array are shown in Fig. 6. Numerically, the computed pattern has an average error of 4.16 dB when compared to the desired beam pattern, with most of the error being attributable to the sidelobe region. The element excitations and the resulting pattern are computed for $f = 2.9$ GHz, since the antenna elements used in the measurements have a center frequency

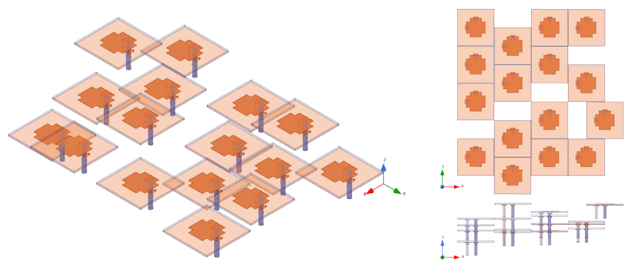

FIGURE 7. **The HFSS model of the 16-element arbitrary array – isometric (left), top (top right), and side (bottom right) views.**

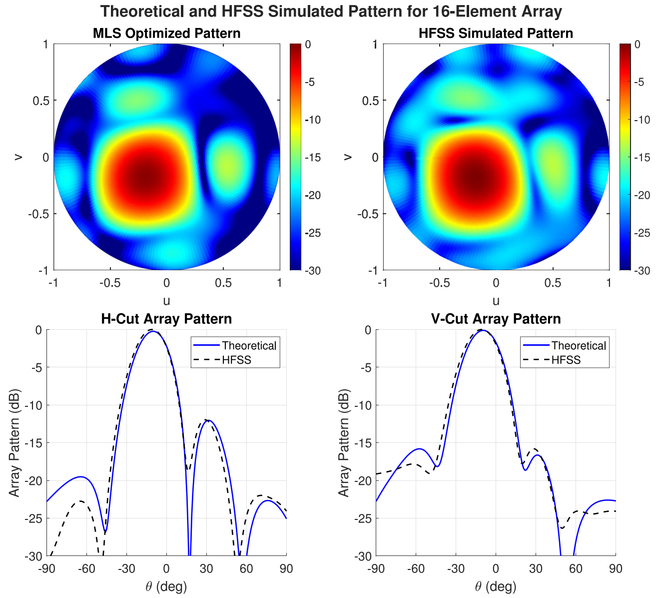

FIGURE 8. **The theoretical synthesized array pattern and the HFSS simulated pattern for the 16-element array, in the full (u, v) space and in H- and V-cuts.**

of 2.9 GHz. As discussed above, due to the lower number of elements, the pattern quality is slightly better when the initial weights are uniformly unitary rather than set to the arbitrary Woodward-Lawson weights, so in this case these weights are used instead. Note that the optimized patterns shown in Fig. 6 are multiplied by the element pattern of the patch antenna, leading to additional mismatch in the sidelobes between the realized pattern and the desired pattern.

B. FULL-WAVE EM SIMULATION

The array is simulated in Ansys HFSS with the element positions defined in Table 2. Each element is an identical coaxial probe-fed patch antenna placed on a 5 cm × 5 cm substrate with a center frequency of 2.9 GHz. Images of the simulation model are given in Fig. 7. The pattern results, compared to the theoretical pattern, are shown in Fig. 8. Numerically, the HFSS simulated pattern has an average error of 3.82 dB when compared to the ideal desired beam pattern, with most of the error being attributable to the sidelobe region. The HFSS simulated results match closely to the theoretical pattern, with the main beam of the pattern matching nearly perfectly and

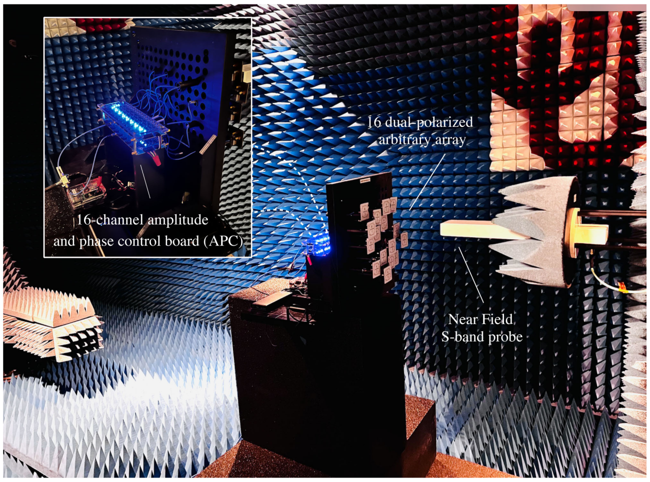

FIGURE 9. **Photo of the antenna array setup at the Radar Innovations Lab (RIL) Near-Field chamber. In the top left, a zoom-in of the 16-channel amplitude and phased controller board.**

some mismatch in the sidelobes. The mismatch is likely attributed to the difference between the single patch antenna element pattern used in the theoretical pattern and the true embedded element pattern for each element induced by the atypical geometry of the array.

C. NEAR-FIELD MEASUREMENTS

The arbitrary patch array modeled in Fig. 7 is constructed using 16 individual dual-polarized square microstrip patch antennas fabricated to match those simulated in the HFSS simulation. The antennas are fastened to rods which can be inserted into holes of a mounting board drilled in a grid pattern with 2.5 cm spacing between the holes, allowing the antennas to be moved in the x- and y-dimension to arbitrary locations with 2.5 cm resolution. The rods can be pulled in or out of the holes to arbitrary distances, giving a continuous range of motion in the z-dimension. The antennas are arranged to match the geometry in Table 2.

Each antenna is connected via coaxial cable to one of two custom 8-channel phased array control boards with independent phase and amplitude shifters on each channel. The phase shifters have a resolution of 5.625° and the amplitude shifters have a resolution of 0.5 dB, so the phase and amplitude values listed in Table 2 are rounded to the nearest increment. Fig. 9 shows the 16-element arbitrary array setup and integration with the APC module using coaxial cables.

Before programming the phase and amplitude shifters, each element is calibrated to determine its constant relative phase and amplitude offset. This is done by connecting each element individually to one port of a 2-port VNA and connecting another calibration antenna to the other port. Using a dielectric positioning tool, the calibration antenna is placed 2.5λ from the antenna element to be calibrated, and the amplitude and phase are recorded. After each element in the array is measured in this way, the results are normalized and backed out of the amplitude and phase values in Table 2.

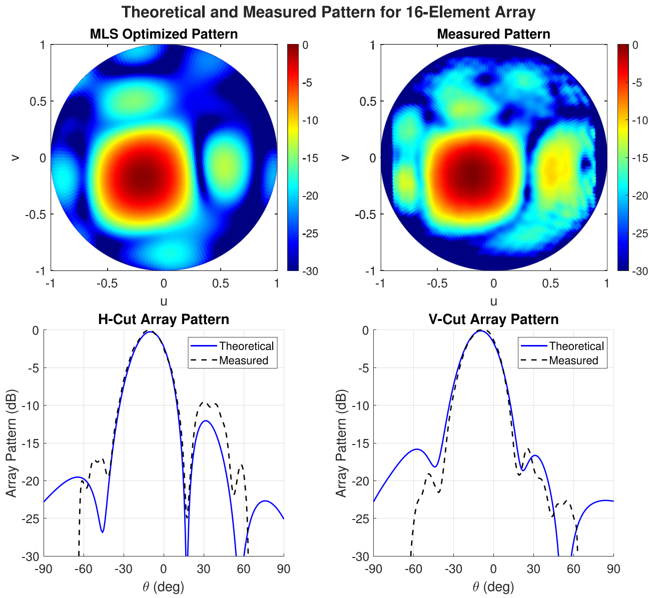

FIGURE 10. The theoretical synthesized array pattern and the measured pattern for the 16-element array, in the full (u, v) space and in H- and V-cuts.

After programming the array controller with the correct amplitude and phase values, the array pattern is measured using a near-field scanner located in an anechoic chamber. An image of this measurement setup is also given in Fig. 9. The final array pattern is obtained via a transformation from the near-field pattern to the far-field pattern. The final measured pattern is compared to the theoretical array pattern in Fig. 10. Numerically, the computed pattern has an average error of 4.01 dB when compared to the desired beam pattern, with most of the error being attributable to the sidelobe region.

The measured results match very well to the theoretical pattern and demonstrate that the proposed algorithm is effective for developing physical systems. As with the HFSS simulated results, the main lobe of the measured pattern matches well to the theoretical pattern, with the main deviations from the expectation occurring in the sidelobes of the pattern. Note that due to the finite extent of the near-field scanner, the computation of the far-field pattern from the near-field data is restricted to $-60^\circ \leq \theta \leq 60^\circ$. The variation between the patterns may be attributed to the mismatch between the element pattern used in the computation of the theoretical results and the actual embedded element pattern, similar to the HFSS simulation error. Additionally, there will be some error due to imprecision in the placement and measurement of the element locations, element-level calibration errors, and rounding errors in the digital amplitude and phase shifters.

V. CONCLUSION

In this paper, an algorithm for synthesizing element excitations to form shaped beam patterns using non-periodic arrays of arbitrary geometry is presented. The algorithm makes no assumptions about the placement of the elements, enabling arrangement in all three spatial dimensions, and it also enables the generation of two-dimensional patterns, allowing for the shaped beam pattern to be any function of u and v. Utilizing a combination of the magnitude least-squares optimization procedure and a novel extension of the Woodward-Lawson beam pattern synthesis technique, the algorithm is able to produce element excitations to form the desired beam shape within a short period of time even for a large number of elements. This makes it ideal for use in real-time applications when the beam shape must be changed on-line, and may also be used in distributed systems with mobile elements when the array geometry itself is not constant. The description of the algorithm is provided, along with multiple examples to demonstrate its function. Finally, the algorithm is demonstrated in practical phased array hardware in both a full-wave EM simulation and with a fabricated phased array system with randomly located elements.

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